

Table of Contents

	Preface	vii
	Basic Terminology	xxix
1	Matchings in bipartite graphs	
1.0.	Introduction	1
1.1.	The Theorems of König, P. Hall and Frobenius	4
	Box 1A. NP-properties, good characterizations and minimax theorems	8
1.2.	A bipartite matching algorithm: the Hungarian Method	12
	Box 1B. On algorithms	16
1.3.	Deficiency, surplus and a glimpse of matroid theory	17
	Box 1C. Matroids	27
1.4.	Some consequences of bipartite matching theorems	29
2	Flow theory	
2.0.	Introduction	41
2.1.	The Max-Flow Min-Cut Theorem	42
2.2.	Flow algorithms	46
	Box 2A. Searching a graph	53
	Box 2B. Numbers in algorithms	59
2.3.	Flow-equivalent trees	61
2.4.	Applications of flow theory to matching theory	68
2.5.	Matchings, flows and measures	74
3	Size and structure of maximum matchings	
3.0.	Introduction	83
3.1.	Tutte's theorem, Gallai's lemma and Berge's formula	84
	Box 3A. Matching matroids and matroid duality	92

3.2.	The Gallai-Edmonds Structure Theorem	93
3.3.	Toward a calculus of barriers	102
3.4.	Sufficient conditions for matchings of a given size	110
4	Bipartite graphs with perfect matchings	
4.0.	Introduction	121
4.1.	Elementary graphs and their ear structure	122
4.2.	Minimal elementary bipartite graphs	127
4.3.	Decomposition into elementary bipartite graphs	137
5	General graphs with perfect matchings	
5.0.	Introduction	143
5.1.	Elementary graphs: elementary properties	145
5.2.	The canonical partition $\mathcal{P}(G)$	150
5.3.	Saturated graphs and cathedrals	159
5.4.	Ear structure of 1-extendable graphs	174
5.5.	More about factor-critical and bicritical graphs	195
6	Some graph-theoretical problems related to matchings	
6.0.	Introduction	213
6.1.	2-matchings and 2-covers	213
6.2.	2-bicritical and regularizable graphs	217
6.3.	Matchings, 2-matchings and the König Property	220
	Box 6A. Reducibility problems and NP-completeness	226
6.4.	Hamilton cycles and 2-matchings	228
6.5.	The Chinese Postman Problem	231
6.6.	Optimum paths, cycles, joins and cuts	243
	Box 6B. Packing paths, cycles, joins and cuts	253

7	Matching and linear programming	
7.0.	Introduction	255
	Box 7A. Cones, polytopes and polyhedra, and other preliminaries from linear programming	256
	Box 7B. Linear programming algorithms	261
7.1.	Linear programming and matching in bigraphs	266
	Box 7C. The Hoffman-Kruskal Theorem and other conditions of integrality	272
7.2.	Matchings and fractional matchings	273
7.3.	The matching polytope	274
	Box 7D. Cutting planes	283
7.4.	Chromatic index	285
	Box 7E. Good characterizations other than the Farkas Lemma	290
7.5.	Fractional matching polytopes and cover polyhedra	291
7.6.	The dimension of the perfect matching polytope	292
	Box 7F. The dimension of a "good" polytope	303
8	Determinants and matchings	
8.0.	Introduction	307
8.1.	Permanents	309
8.2.	The method of variables	315
8.3.	The Pfaffian and the number of perfect matchings	318
8.4.	Probabilistic enumeration of perfect matchings	330
	Box 8A. Probabilistic methods in graph theory	331
8.5.	Matching polynomials	333
8.6.	More on the number of perfect matchings	345
8.7.	Two applications to physical science	349

9	Matching algorithms	
9.0.	Introduction	357
9.1.	The Edmonds Matching Algorithm	358
9.2.	Weighted matching	369
9.3.	An algorithm based upon the Gallai-Edmonds Theorem	376
9.4.	A linear programming algorithm for matching	379
10	The f-factor problem	
10.0.	Introduction	383
10.1.	Reduction principles	385
10.2.	A structure theory for f -factors	388
10.3.	Realization of degree sequences	404
11	Matroid matching	
11.0.	Introduction	409
11.1.	Formulations of the Matroid Matching Problem	409
	Box 11A. Oracles	413
	Box 11B. Minimizing submodular set functions	417
11.2.	The main theorem of polymatroid matching	426
11.3.	Matching in special polymatroids	433
12	Vertex packing and covering	
12.0.	Introduction	443
12.1.	Critical graphs	445
12.2.	Vertex packing polytopes	456
12.3.	Hypergraph matching	466
12.4.	Vertex packing in claw-free graphs	471
	Box 12A. Bounds on the independence number, or: can anything be done with NP-complete problems?	480

**Appendix: Developments in matching theory since
this book was first published**

A1	The matching structure	A1
A1.1	A calculus of barriers	A1
A1.2	Brick and brace decomposition	A1
A1.3	Removable lines	A2
A1.4	Ear-decompositions	A2
A1.5	The matching lattice	A3
A1.6	Equimatchable graphs	A4
A1.7	Extendability	A4
A2	Sampling and counting matchings	A6
A2.1	The number of matchings in regular bipartite graphs	A6
A2.2	Monte-Carlo sampling and approximate counting	A6
A2.3	Properties of random matchings	A8
A3	Improvements on the determinant method	A9
A3.1	Explicit substitution on the Tutte matrix	A9
A3.2	Substituting random exponentials	A9
A3.3	Random substitutions and the number of perfect matchings	A10
A3.4	Pfaffian graphs	A10
A4	Matching in parallel	A11
A5	Facets of the stable set polytope and α -critical graphs	A12
A6	Factorization	A13
A7	Structures generalizing matchings and matroids	A13
A7.1	Path matching	A13
A7.2	Jump systems	A14
	References	A15
	References	483
	Index of Terms	527
	Index of Symbols	539
	Errata in the book	545