

# Contents

<b>Mathematical Modelling of Glass Forming Processes</b> .....	1	
J.A.W.M. Groot, Robert M.M. Mattheij, and K.Y. Laevsky		
1	Introduction .....	2
1.1	Glass Forming .....	2
1.2	Process Simulation .....	6
1.3	Outline .....	7
2	Mathematical Model .....	8
2.1	Geometry, Problem Domains and Boundaries .....	8
2.2	Balance Laws .....	9
3	Parison Press Model .....	16
3.1	Mathematical Model .....	16
3.2	Slender-Geometry Approximation .....	18
3.3	Motion of the Plunger .....	23
3.4	Simulation Model .....	26
3.5	Results .....	30
4	Blow Model .....	31
4.1	Mathematical Model .....	32
4.2	Glass-Air Interfaces .....	36
4.3	Variational Formulation .....	39
4.4	Simulation Model .....	42
4.5	Results .....	45
5	Direct Press Model .....	46
5.1	Mathematical Model .....	47
5.2	Simulation Model .....	49
5.3	Results .....	51
References .....		53
<b>Radiative Heat Transfer and Applications for Glass Production Processes</b> .....		57
Martin Frank and Axel Klar		
1	Introduction .....	57
2	Radiative Heat Transfer Equations for Glass .....	58
2.1	Fundamental Quantities .....	59

2.2	Blackbody Radiation .....	61
2.3	The Transfer Equation.....	62
2.4	Overall Energy Conservation .....	65
2.5	Boundary Conditions.....	66
2.6	Summary .....	68
3	Direct Numerical Methods .....	70
3.1	Ordinates and Space Discretizations .....	71
3.2	Linear System Formulation .....	73
3.3	Preconditioning Techniques.....	77
3.4	A Fast Multilevel Preconditioner .....	82
3.5	Numerical Results .....	85
4	Higher-Order Diffusion Approximations.....	92
4.1	Asymptotic Analysis and Derivation of the $SP_N$ Approximations .....	94
4.2	Boundary Conditions for $SP_N$ Approximations.....	100
5	Moment Models .....	104
5.1	Spherical Harmonics .....	105
5.2	Minimum Entropy Closure.....	107
5.3	Flux-Limited Diffusion and Entropy Minimization .....	108
5.4	Partial Moments .....	110
5.5	Partial Moment $P_N$ Closure .....	112
5.6	Partial Moment Entropy Closure .....	113
6	Frequency-Averaged Moment Equations.....	115
6.1	Entropy Minimization.....	116
6.2	Inversion of the System .....	117
6.3	Properties .....	118
7	Numerical Comparisons.....	119
7.1	Numerical Results .....	119
7.2	Grey Transport.....	119
7.3	Grey Cooling .....	120
7.4	Multigroup Transport.....	123
7.5	Multigroup Cooling .....	125
7.6	Adaptive methods for the Simulation of 2-d and 3-d Cooling Processes.....	125
	References .....	131

## **Radiative Heat Transfer and Applications for Glass Production**

	<b>Processes II .....</b>	<b>135</b>
	Norbert Siedow	
1	Introduction .....	135
2	Models for Fast Radiative Heat Transfer Simulation .....	137
2.1	Introduction .....	137
3	Indirect Temperature Measurement of Hot Glasses.....	148
3.1	Introduction .....	148

- 3.2 The Basic Equation of Spectral Remote Temperature Sensing ..... 149
- 3.3 Some Basics of Inverse Problems ..... 150
- 3.4 Spectral Remote Sensing ..... 159
- 3.5 Reconstruction of Initial Temperature ..... 161
- 3.6 Conclusions ..... 170
- References ..... 170

**Non-Isothermal Flow of Molten Glass: Mathematical**

**Challenges and Industrial Questions ..... 173**

Angiolo Farina, Antonio Fasano, and Andro Mikelić

- 1 Introduction ..... 173
- 2 Mathematical Modelling ..... 176
  - 2.1 Definitions and Basic Equations ..... 176
  - 2.2 Fluids Physical Properties and Constitutive Equations ..... 177
  - 2.3 The General Model ..... 181
  - 2.4 Scaling and Dimensionless Formulation ..... 183
- 3 Study of the Stationary Non-Isothermal Molten Glass Flow in a Die .... 187
  - 3.1 Existence and Uniqueness Result for the Stationary Problem... 189
  - 3.2 Oberbeck–Boussinesq Model ..... 194
- 4 Modelling the Viscous Jet at the Exit of the Die ..... 198
  - 4.1 Definition of  $L$  and Jet’s Profile at the End of Stage ( $c$ ) ..... 201
- 5 Terminal Phase of the Fiber Drawing ..... 207
  - 5.1 Derivation of the Model of Matovich–Pearson for the Thermal Case ..... 209
  - 5.2 Solvability of the Boundary Value Problems for the Stationary Effective Equations ..... 216
- References ..... 223

**List of Participants ..... 225**