

# Contents

Figure acknowledgements	xvi
Abbreviations in Reference Lists	xx
<b>1 Introduction</b>	1
1.1 Historical survey	1
1.2 Stellar magnetic fields	6
Bibliography	11
<b>2 Theoretical basis</b>	13
2.1 Maxwell's equations and the magnetohydrodynamic approximation	13
2.2 Properties of cosmical plasmas	16
2.3 Macroscopic equations for a fully ionized gas: the two-fluid model	19
2.3.1 Equations to the flow of the whole gas	20
2.3.2 The generalized Ohm's law	23
2.4 The energy equation of a fully ionized gas	27
2.5 Kinematic coupling	29
2.6 Dynamical coupling	33
2.7 The three-fluid model	37
2.8 'Anomalous' resistivity	40
Bibliography	43
<b>3 Applications</b>	45
3.1 Magnetosonic waves	45
3.2 Magnetohydrodynamic shocks	47
3.3 Self-gravitating systems: the virial theorems	52
3.4 Magnetostatic equilibrium: force-free fields	58
3.5 Magnetic helicity	65
3.6 Stability	69
3.6.1 The MHD energy principle	69
3.6.2 Illustrative examples	71
3.6.3 The pinched cylindrical discharge	74
3.6.4 The Kelvin–Helmholtz instability	79
3.6.5 Stability of rotating systems	82
3.7 Effects of dissipation: reconnection	84
3.7.1 Reconnection in a medium at rest	85
3.7.2 The Sweet–Parker model	88
3.7.3 Fast reconnection	91
3.7.4 Hall reconnection	95
3.8 Macroscopic dissipation	97
Appendix: Poloidal and toroidal fields	99
Bibliography	100

<b>4 Magnetism and convection</b>	104
4.1 Introduction	104
4.2 The angular velocity distribution in a convective zone	108
4.2.1 The Reynolds stresses	108
4.2.2 Departure from adiabaticity	111
4.3 The effect of convective motions on an imposed magnetic field	115
4.4 A strong imposed field and the onset of convection	120
4.4.1 Imposed field vertical	120
4.4.2 Imposed field horizontal	123
4.5 Non-linear theory: recent developments	124
4.5.1 The non-magnetic problem	125
4.5.2 Magnetoconvection	129
4.6 Sunspots, pores, and isolated flux tubes	134
4.7 Magnetic buoyancy	137
4.7.1 Flux tubes	137
4.7.2 Instability in magnetically supported domains	139
4.7.3 Non-linear developments	140
4.8 Solar activity	141
4.8.1 Flux tube dynamics	142
4.8.2 Chromospheric and coronal MHD	148
Bibliography	153
<b>5 Magnetic fields in stellar interiors</b>	160
5.1 General considerations	160
5.2 Magnetic fields and stellar rotation	162
5.2.1 Axisymmetric states	162
5.2.2 ‘Quasi-steady’, non-axisymmetric states	166
5.3 Stability	167
5.4 Laminar meridional flow in radiative domains	171
5.5 The interaction between rotation, magnetism, and circulation	176
5.5.1 Steady-state integrals	176
5.5.2 Equatorial acceleration	179
5.5.3 The approach to a quasi-steady state	180
5.6 Ohmic decay of primeval magnetic fields	183
5.6.1 Decay of a purely poloidal field	183
5.6.2 Decay of a mixed poloidal–toroidal field	185
5.7 The Biermann ‘battery’ process	188
5.7.1 Coupling with a poloidal field	189
5.7.2 The effect of chemical inhomogeneities	191
5.8 An introduction to the stellar dynamo problem	193
5.8.1 Cowling’s anti-dynamo theorem	193
5.8.2 Mass motions and the rate of decay	196
Bibliography	198

<b>6</b>	<b>Dynamo processes in stars</b>	201
6.1	Introduction	201
6.2	Laminar kinematic dynamos	203
6.3	The Parker model	207
6.4	Turbulent dynamos	210
6.4.1	Mean-field electrodynamics: the classical treatment	210
6.4.2	Isotropic turbulence	217
6.4.3	Kinematics and dynamics in the low Reynolds number domain	219
6.5	Kinematic models of the turbulent dynamo	222
6.5.1	General discussion	222
6.5.2	The $\alpha\Omega$ dynamo	224
6.5.3	A model with separate shear and $\alpha$ -effect zones	225
6.6	Non-linear dynamical feedback	230
6.6.1	Buoyancy-limited growth	230
6.6.2	Magnetic back-reaction: modulated cycles	232
6.7	Fundamental problems	236
6.7.1	The $\alpha$ -effect and helicity	236
6.7.2	The dynamical back-reaction	238
6.7.3	Analytical treatment	239
6.8	The role of magnetic helicity in the dynamo problem	241
6.8.1	Magnetic helicity evolution	241
6.8.2	Dynamical $\alpha$ -quenching in closed or periodic domains	243
6.8.3	Mean-field models with magnetic helicity flux	245
6.9	Numerical simulations	246
6.9.1	$\alpha$ - and $\eta$ -quenching	246
6.9.2	Further numerical work: a return to first principles	248
6.10	Dynamo action guided by a strong pre-existing field	250
6.10.1	A dynamo driven by the instability of strong flux tubes	251
6.10.2	A two-dimensional flux tube model	254
6.11	Conclusions	256
	Bibliography	257
<b>7</b>	<b>Stellar winds: magnetic braking</b>	263
7.1	Introduction	263
7.2	The braking of axisymmetric systems	265
7.3	The wind theory	267
7.4	The structure of the poloidal field	270
7.4.1	General discussion	270
7.4.2	A numerical attack	273
7.4.3	Asymptotic behaviour	276
7.5	A simple field model	280
7.6	The rate of braking	284
7.7	A digression on the micro-physics	286

7.8	Magnetic braking of the oblique rotator	289
7.8.1	The generalized wind theory	289
7.8.2	The gross dynamics of the star	292
7.8.3	The effect of the thermo-centrifugal wind	293
7.9	Winds driven by Alfvén waves	295
7.10	The solar wind revisited	301
7.11	Radiation-driven winds from hot early-type stars	304
	Appendix A: Alfvén waves in a multi-component plasma	307
	Appendix B: The axisymmetric magnetic rotator: the energetics	308
	Bibliography	310
<b>8</b>	<b>Late-type stars</b>	<b>313</b>
8.1	Introduction	313
8.2	The ‘solar–stellar connection’	315
8.2.1	The rapidly rotating dwarf star AB Doradus	319
8.3	The rotational history of late-type main-sequence stars	322
8.4	The Sun: new observational material	330
8.4.1	Solar activity: the solar cycle	330
8.4.2	The internal solar rotation	332
8.5	Phenomenological studies of the solar dynamo	336
8.6	The solar dynamo revisited	339
8.6.1	Historical résumé	339
8.6.2	Current ideas on the solar dynamo	343
8.7	Further recent computations	345
8.7.1	The rotation of the convective envelope	345
8.7.2	Dynamo action	351
8.8	The tachocline	355
8.8.1	Non-magnetic theory	355
8.8.2	Subsequent developments: gyroscopic pumping and magnetohydrodynamic theory	358
8.8.3	A slow tachocline dynamo	363
8.8.4	Application to tachocline dynamics	368
8.8.5	The ‘Li problem’	370
8.8.6	Résumé	371
8.8.7	Subsequent developments	373
8.9	The solar–stellar connection revisited	375
8.9.1	Sub-solar-mass stars	375
8.9.2	Young solar-mass stars	377
8.10	Return to the standard dynamo equations	379
8.10.1	Modulation of cyclic activity	380
8.10.2	Rapidly rotating late-type stars	387
8.10.3	Evolved stars	392
8.10.4	Non-axisymmetric field generation	394
	Bibliography	396

<b>9</b>	<b>The early-type magnetic stars</b>	407
9.1	The basic observational data: historical summary	407
9.1.1	Field-structure modelling	409
9.1.2	Correlations	412
9.1.3	Evolution	413
9.1.4	Problems	414
9.2	Stability of large-scale stellar magnetic fields	415
9.3	The dynamics of the oblique rotator: the Eulerian nutation and the consequent internal motions	417
9.3.1	The construction of a unique $\xi$ -field	421
9.3.2	Consequences of the $\xi$ -motions	424
9.3.3	Dissipation of the $\xi$ -motions	425
9.4	Non-uniform rotation and the oblique rotator model	428
9.5	Models of rotating magnetic stars	431
9.5.1	Axisymmetric radiative zones	432
9.5.2	Models with thermally-driven circulation	433
9.5.3	Generalizations	435
9.6	Magnetic torques acting on the oblique rotator: spin-down, spin-up, and changes in obliquity	438
9.6.1	Braking processes in the pre-main-sequence and main-sequence epochs	438
9.6.2	Changes in obliquity	442
9.7	The origin of the field	449
9.7.1	Recapitulation	453
9.8	Abundance anomalies	456
9.9	The roAp phenomenon	460
	Appendix A: Stellar atmospheres	463
	A1 The atmospheres of non-magnetic stars	463
	A2 Magnetic star atmospheres	466
	Appendix B: Evolution of a dynamically stable magnetic field: an analytical treatment	472
	Bibliography	489
	General references	495
<b>10</b>	<b>Pre-main-sequence stars</b>	496
10.1	The later stages of star formation	496
10.2	Magnetic accretion discs	501
10.2.1	The magnetosphere	502
10.2.2	Canonical disc theory: angular momentum transport	504
10.2.3	An illustrative model	506
10.2.4	The estimated net torque	508
10.3	Pre-main-sequence rotational evolution	510
10.4	Later developments	516
10.4.1	X-ray observations	516
10.4.2	Accretion disc theory: later developments	517

10.4.3	Models with reduced magnetic coupling between star and disc	518
10.4.4	Numerical simulations	520
10.4.5	Disc locking	521
10.5	Instability in a magnetic rotating disc	522
10.5.1	The magneto-rotational instability	523
10.5.2	A more formal treatment	526
10.5.3	Angular momentum transport in a thin radiative disc	530
10.6	Disc dynamos	532
10.6.1	Applications of the ‘standard dynamo equations’	532
10.6.2	Dynamo action driven by the magneto-rotational instability	536
10.6.3	Comments and queries	538
10.7	Centrifugal winds from discs	538
10.7.1	Cold, centrifugally-driven winds	539
10.7.2	The flow near the disc surface	542
10.8	Collimation	545
10.8.1	Toroidal field collimation	545
10.8.2	Detailed models	548
10.8.3	Collimation by the poloidal field	554
10.9	Conclusion	557
	Appendix A: The model of Section 10.2: canonical disc theory	557
	Appendix B: Other instabilities in discs	564
	Bibliography	569
<b>11</b>	<b>Magnetism and star formation I</b>	<b>576</b>
11.1	Introduction	576
11.2	Magneto-thermo-gravitational equilibrium	580
11.2.1	A spherical cloud model	580
11.2.2	A spheroidal model	583
11.2.3	Résumé	589
11.3	Applications	589
11.3.1	The accumulation length	589
11.3.2	The $B-\rho$ relations in a cool cloud	591
11.3.3	Strongly turbulent clouds	594
11.4	Gravitational collapse under flux-freezing: possible fragmentation	595
11.5	The angular momentum problem	601
11.6	Magnetic braking by Alfvén waves	605
11.6.1	An axisymmetric cylindrical model	605
11.6.2	Braking by a radially distorted field	608
11.6.3	Magnetic braking and gravitational contraction	610
11.6.4	A perpendicular magnetic rotator	612
11.6.5	Fragmentation of a rotating magnetic cloud	613

11.7	Flux leakage	614
11.7.1	Ambipolar diffusion	614
11.7.2	Quasi-steady contraction of an oblate spheroidal model	619
	Appendix A: The model of Figure 11.2	623
	Appendix B: Magnetic braking by Alfvén waves: detailed treatment	627
<b>12</b>	<b>Magnetism and star formation II</b>	<b>636</b>
12.1	Résumé	636
12.2	Magneto-gravitational equilibrium: exact disc-like models	640
12.2.1	Finite disc models	640
12.2.2	Infinite disc models	645
12.2.3	Collapsed core models	648
12.2.4	Disc models with partial turbulent support	650
12.2.5	Magneto-gravitational equilibrium: summary	653
12.3	Magneto-turbulent cloud models	654
12.4	Evolution through flux diffusion	657
12.5	Gravitational collapse	660
12.6	Field line detachment	661
12.7	Flux leakage	663
12.8	Magnetic ‘levitation’?	664
12.9	Alfvénic turbulence	671
12.9.1	Non-dissipative theory	671
12.9.2	The effect of dissipation	676
12.10	Turbulent ambipolar diffusion	679
12.11	The future	680
12.11.1	Magneto-turbulence and star formation	680
12.12	Summary	684
	Appendix A: Exact disc-like models	687
	Appendix B: Magnetized singular isothermal toroids	692
	Appendix C: Isopedic disc models	696
	Appendix D: Turbulent ambipolar diffusion	699
	Bibliography (Chapters 11 and 12)	704
	<b>Index</b>	<b>710</b>