

Contents

Preface to the Third Edition	v
Preface to the Second Edition	vii
Preface	ix
Contents	xi
Introduction	xv
CHAPTER 1	
Random Perturbations	1
1 Probabilities and Random Variables	1
2 Random Processes. General Properties	3
3 Wiener Process. Stochastic Integral	9
4 Markov Processes and Semigroups	15
5 Diffusion Processes and Differential Equations	19
CHAPTER 2	
Small Random Perturbations on a Finite Time Interval	29
1 Zeroth Order Approximation	29
2 Expansion in Powers of a Small Parameter	36
3 Elliptic and Parabolic Differential Equations with a Small Parameter .	44
CHAPTER 3	
Action Functional	54
1 Laplace's Method in a Function Space	54
2 Exponential Estimates	57
3 Action Functional. General Properties	63
4 Action Functional for Gaussian Random Processes and Fields	75

CHAPTER 4**Gaussian Perturbations of Dynamical Systems. Neighborhood of an Equilibrium Point** 85

1	Action Functional	85
2	The Problem of Exit from a Domain	89
3	Properties of the Quasipotential. Examples	100
4	Asymptotics of the Mean Exit Time and Invariant Measure	105
5	Gaussian Perturbations of General Form	114

CHAPTER 5**Perturbations Leading to Markov Processes** 117

1	Legendre Transformation	117
2	Locally Infinitely Divisible Processes	124
3	Special Cases. Generalizations	134
4	Consequences. Generalization of Results of Chap. 4	137

CHAPTER 6**Markov Perturbations on Large Time Intervals** 142

1	Auxiliary Results. Equivalence Relation	142
2	Markov Chains Connected with the Process $(X_t^\varepsilon, P_x^\varepsilon)$	150
3	Lemmas on Markov Chains	157
4	The Problem of the Invariant Measure	165
5	The Problem of Exit from a Domain	172
6	Decomposition into Cycles. Metastability	178
7	Eigenvalue Problems	184

CHAPTER 7**The Averaging Principle. Fluctuations in Dynamical Systems with Averaging** 192

1	The Averaging Principle in the Theory of Ordinary Differential Equations	192
2	The Averaging Principle when the Fast Motion is a Random Process	196
3	Normal Deviations from an Averaged System	198
4	Large Deviations from an Averaged System	212
5	Large Deviations Continued	219
6	The Behavior of the System on Large Time Intervals	226
7	Not Very Large Deviations	230
8	Examples	235
9	The Averaging Principle for Stochastic Differential Equations	244

CHAPTER 8**Random Perturbations of Hamiltonian Systems** 258

1	Introduction	258
2	Main Results	269
3	Proof of Theorem 2.2	275

4	Proof of Lemmas 3.1 to 3.4	285
5	Proof of Lemma 3.5	300
6	Proof of Lemma 3.6	311
7	Remarks and Generalizations	316
8	Deterministic Perturbations of Hamiltonian Systems. One Degree of Freedom	332

CHAPTER 9

The Multidimensional Case	355
1 Slow Component Lives on an Open Book Space	355
2 The Results Outside the Singularities	360
3 Weakly Coupled Oscillators. Formulation of the Results	367
4 The Markov Process $(Y(t), P_y)$ on Γ : Existence and Uniqueness	372
5 Proof of Theorem 3.2	376
6 Deterministic Coupling	384

CHAPTER 10

Stability Under Random Perturbations	390
1 Formulation of the Problem	390
2 The Problem of Optimal Stabilization	396
3 Examples	401

CHAPTER 11

Sharpenings and Generalizations	405
1 Local Theorems and Sharp Asymptotics	405
2 Large Deviations for Random Measures	412
3 Processes with Small Diffusion with Reflection at the Boundary	419
4 Wave Fronts in Semilinear PDEs and Large Deviations	423
5 Random Perturbations of Infinite-Dimensional Systems	433

References	441
-------------------	-----

Index	457
--------------	-----