

# Contents

## PART I

### THE STATISTICAL ANALYSIS OF COINTEGRATION

1	Introduction	3
1.1	<i>The vector autoregressive model</i>	4
1.2	<i>Building statistical models</i>	5
1.3	<i>Illustrative examples</i>	7
1.4	<i>An outline of the contents</i>	8
1.5	<i>Some further problems</i>	10
2	The Vector Autoregressive Model	11
2.1	<i>The vector autoregressive process</i>	11
2.2	<i>The statistical analysis of the VAR</i>	17
2.3	<i>Misspecification tests</i>	20
2.4	<i>The illustrative examples</i>	23
3	Basic Definitions and Concepts	34
3.1	<i>Cointegration and common trends</i>	34
3.2	<i>Exercises</i>	42
4	Cointegration and Representation of Integrated Variables	45
4.1	<i>From AR to MA representation for <math>I(1)</math> variables</i>	45
4.2	<i>From MA to AR representation for <math>I(1)</math> variables</i>	55
4.3	<i>The MA representation of <math>I(2)</math> variables</i>	57
4.4	<i>Exercises</i>	62
5	The $I(1)$ Models and their Interpretation	70
5.1	<i>The <math>I(1)</math> models for cointegration</i>	70
5.2	<i>The parametrization of the <math>I(1)</math> model</i>	71
5.3	<i>Hypotheses on the long-run coefficients <math>\beta</math></i>	72
5.4	<i>Hypotheses on the adjustment coefficients <math>\alpha</math></i>	77
5.5	<i>The structural error correction model</i>	78
5.6	<i>General hypotheses</i>	79
5.7	<i>Models for deterministic terms</i>	80

5.8	<i>Intervention and seasonal dummies</i>	84
5.9	<i>Exercises</i>	85
6	The Statistical Analysis of $I(1)$ Models	89
6.1	<i>Likelihood analysis of <math>H(r)</math></i>	89
6.2	<i>Models for the deterministic terms</i>	95
6.3	<i>Determination of cointegrating rank</i>	98
6.4	<i>Exercises</i>	100
7	Hypothesis Testing for the Long-Run Coefficients $\beta$	104
7.1	<i>Degrees of freedom</i>	104
7.2	<i>Linear restrictions on <math>\beta</math></i>	106
7.3	<i>Illustrative examples</i>	112
7.4	<i>Exercises</i>	120
8	Partial Systems and Hypotheses on $\alpha$	121
8.1	<i>Partial systems</i>	121
8.2	<i>Test of restrictions on <math>\alpha</math></i>	124
8.3	<i>The duality between <math>\hat{\beta}</math> and <math>\hat{\alpha}_\perp</math></i>	128
8.4	<i>Exercises</i>	131
9	The $I(2)$ Model and a Test for $I(2)$	132
9.1	<i>A statistical model for <math>I(2)</math></i>	132
9.2	<i>A misspecification test for the presence of <math>I(2)</math></i>	134
9.3	<i>A test for <math>I(2)</math> in the Australian data</i>	137

## PART II

### THE PROBABILITY ANALYSIS OF COINTEGRATION

10	Probability Properties of $I(1)$ Processes	141
10.1	<i>Finite sample results</i>	141
10.2	<i>Asymptotic results</i>	143
10.3	<i>Exercises</i>	149
11	The Asymptotic Distribution of the Test for Cointegrating Rank	151
11.1	<i>Testing <math>\Pi = 0</math> in the basic model</i>	151
11.2	<i>The limit distribution of the test for cointegrating rank</i>	156
11.3	<i>Asymptotic properties of the test for <math>I(2)</math></i>	163
11.4	<i>Exercises</i>	164

12	Determination of Cointegrating Rank	167
12.1	<i>Model without constant term</i>	168
12.2	<i>Model with a constant term</i>	170
12.3	<i>Models with a linear term</i>	175
12.4	<i>Exercises</i>	176
13	Asymptotic Properties of the Estimators	177
13.1	<i>The mixed Gaussian distribution</i>	177
13.2	<i>A convenient normalization of <math>\beta</math></i>	179
13.3	<i>Consistency of the estimators</i>	180
13.4	<i>Asymptotic distribution of <math>\hat{\beta}</math> and <math>\hat{\alpha}</math></i>	181
13.5	<i>More asymptotic distributions</i>	187
13.6	<i>Likelihood ratio test for hypotheses on the long-run coefficients <math>\beta</math></i>	192
13.7	<i>Exercises</i>	196
14	The Power Function of the Test for Cointegrating Rank under Local Alternatives	201
14.1	<i>Local alternatives</i>	201
14.2	<i>Properties of the process under local alternatives</i>	202
14.3	<i>The local power of the trace test</i>	206
14.4	<i>Exercises</i>	209
15	Simulations and Tables	211
15.1	<i>Simulation of the limit distributions</i>	211
15.2	<i>Simulations of the power function</i>	212
15.3	<i>Tables</i>	214

## PART III

## APPENDICES

A	Some Mathematical Results	219
A.1	<i>Eigenvalues and eigenvectors</i>	219
A.2	<i>The binomial formula for matrices</i>	228
A.3	<i>The multivariate Gaussian distribution</i>	233
A.4	<i>Principal components and canonical correlations</i>	237
B	Weak Convergence of Probability Measures on $\mathbb{R}^p$ and $C[0,1]$	239
B.1	<i>Weak convergence on <math>\mathbb{R}^p</math></i>	239

B.2	<i>Weak convergence on <math>C[0, 1]</math></i>	241
B.3	<i>Construction of measures on <math>C[0, 1]</math></i>	244
B.4	<i>Tightness and Prohorov's theorem</i>	245
B.5	<i>Construction of Brownian motion</i>	247
B.6	<i>Stochastic integrals with respect to Brownian motion</i>	248
B.7	<i>Some useful results for linear processes</i>	250
	References	255
	Subject Index	261
	Author Index	266