Contents

1 Introduction	page l
1.1 Graph theory	4
1.2 Scale-free processes and fractal structures	4
PART I RANDOM NETWORK MODELS	
2 The Erdős–Rényi models	9
2.1 Erdős-Rényi graphs	9
2.2 Scale-free networks	10
2.3 Diameter and fractal dimensions	12
2.4 Random graphs as a model of real networks	13
2.5 Outlook and applications	14
3 Observations in real-world networks: the Internet, epidemics,	
proteins and DNA	16
3.1 Real-world complex networks	16
3.2 Properties of real-world networks	21
3.3 Betweenness centrality: what is your importance in the network?	27
3.4 Conclusions	29
4 Models for complex networks	31
4.1 Introduction	31
4.2 Introducing shortcuts: small-world networks	31
4.3 Random graphs with a given degree distribution	35
4.4 Introducing correlations	39
4.5 Randomly directed networks: modeling the WWW	39
4.6 Introducing geography: embedded scale-free lattices	41
4.7 Hierarchical and fractal networks	48
Exercises	49

ADIAL MUNICIPA

5 Growing network models: the Barabási–Albert model	
and its variants	51
5.1 The Barabási–Albert model	51
5.2 Variants of the Barabási-Albert model	54
5.3 Linearized chord diagram (LCD)5.4 Fitness models	57
	59
Exercises	62
PART II STRUCTURE AND ROBUSTNESS OF COMPLEX NETWORKS	
6 Distances in scale-free networks: the ultra small world	65
6.1 Introduction	65
6.2 Minimal distance networks	6
6.3 Random scale-free networks	69
6.4 Layer structure and Internet tomography – how far do your	
emails travel?	72
6.5 Discussion and conclusions	79
Exercises	80
7 Self-similarity in complex networks	81
8 Distances in geographically embedded networks	88
9 The structure of networks: the generating function method	91
9.1 Introduction	91
9.2 General results	91
9.3 Scale-free networks	94
Exercises	95
10 Percolation on complex networks	91
10.1 Introduction	97
10.2 Random breakdown	9
10.3 Intentional attack	10
10.4 Critical exponents	10:
10.5 Percolation in networks with correlations	114
10.6 k-core percolation: fault tolerant networks	116
10.7 Conclusions	120
Exercises	121

11 Structure of random directed networks: the bow tie	123
11.1 Introduction	123
11.2 Structure	124
11.3 The giant component	124
11.4 Percolation in directed scale-free networks	125
11.5 Critical exponents	128
11.6 Summary	131
Exercises	131
12 Introducing weights: bandwidth allocation and	
multimedia broadcasting	133
12.1 Introduction	133
12.2 Random weighted networks	134
12.3 Correlated weighted networks	140
12.4 Summary	142
Exercises	142
PART III NETWORK FUNCTION: DYNAMICS AND APPLICATIONS	
13 Optimization of the network structure	145
13.1 Introduction	145
13.2 Optimization analysis	146
13.3 General results	149
13.4 Summary	152
14 Epidemiological models	154
14.1 Introduction	154
14.2 Epidemic dynamics and epidemiological models	155
Exercises	160
15 Immunization	161
15.1 Random immunization	161
15.2 Targeted immunization: choosing the right people to immunize	162
15.3 Acquaintance immunization: choosing the right people with	
minimal information	163
15.4 Numerical results for the SIR model	170
15.5 Conclusion	170
Exercises	172

16 Thermodynamic models on networks	173
16.1 Introduction	173
16.2 The Ising model in complex networks	174
16.3 Summary	179
Exercises	180
17 Spectral properties, transport, diffusion and dynamics	181
17.1 The spectrum of the adjacency matrix	181
17.2 The Laplacian	182
17.3 The spectral gap and diffusion on graphs	184
17.4 Traffic and self-similarity	191
17.5 Summary	191
Exercises	192
18 Searching in networks	193
18.1 Introduction	193
18.2 Searching using degrees	193
18.3 Searching in networks using shortcuts	195
18.4 Summary	198
Exercises	199
19 Biological networks and network motifs	200
19.1 Structure of metabolic networks	201
19.2 Structure of genetic networks	201
19.3 Network motifs	201
19.4 Summary	206
Appendix A: Probability theoretical methods	207
Appendix B: Asymptotics and orders of magnitude	213
Appendix C: Algorithms for network simulation and investigation	215
References	222
Index	236