
Contents

Preface	ix
Part 1. Parametric Models	
Chapter 1. The Fisher Efficiency	3
§1.1. Statistical Experiment	3
§1.2. The Fisher Information	6
§1.3. The Cramér-Rao Lower Bound	7
§1.4. Efficiency of Estimators	8
Exercises	9
Chapter 2. The Bayes and Minimax Estimators	11
§2.1. Pitfalls of the Fisher Efficiency	11
§2.2. The Bayes Estimator	13
§2.3. Minimax Estimator. Connection Between Estimators	16
§2.4. Limit of the Bayes Estimator and Minimaxity	18
Exercises	19
Chapter 3. Asymptotic Minimaxity	21
§3.1. The Hodges Example	21
§3.2. Asymptotic Minimax Lower Bound	22
§3.3. Sharp Lower Bound. Normal Observations	26
§3.4. Local Asymptotic Normality (LAN)	28
§3.5. The Hellinger Distance	31
§3.6. Maximum Likelihood Estimator	33

§3.7. Proofs of Technical Lemmas	35
Exercises	40
Chapter 4. Some Irregular Statistical Experiments	43
§4.1. Irregular Models: Two Examples	43
§4.2. Criterion for Existence of the Fisher Information	44
§4.3. Asymptotically Exponential Statistical Experiment	45
§4.4. Minimax Rate of Convergence	47
§4.5. Sharp Lower Bound	47
Exercises	49
Chapter 5. Change-Point Problem	51
§5.1. Model of Normal Observations	51
§5.2. Maximum Likelihood Estimator of Change Point	54
§5.3. Minimax Limiting Constant	56
§5.4. Model of Non-Gaussian Observations	57
§5.5. Proofs of Lemmas	59
Exercises	62
Chapter 6. Sequential Estimators	65
§6.1. The Markov Stopping Time	65
§6.2. Change-Point Problem. Rate of Detection	69
§6.3. Minimax Limit in the Detection Problem.	73
§6.4. Sequential Estimation in the Autoregressive Model	75
Exercises	83
Chapter 7. Linear Parametric Regression	85
§7.1. Definitions and Notations	85
§7.2. Least-Squares Estimator	87
§7.3. Properties of the Least-Squares Estimator	89
§7.4. Asymptotic Analysis of the Least-Squares Estimator	93
Exercises	96
Part 2. Nonparametric Regression	
Chapter 8. Estimation in Nonparametric Regression	101
§8.1. Setup and Notations	101
§8.2. Asymptotically Minimax Rate of Convergence. Definition	103
§8.3. Linear Estimator	104

§8.4. Smoothing Kernel Estimator	106
Exercises	112
Chapter 9. Local Polynomial Approximation of the Regression Function	115
§9.1. Preliminary Results and Definition	115
§9.2. Polynomial Approximation and Regularity of Design	119
§9.3. Asymptotically Minimax Lower Bound	122
§9.4. Proofs of Auxiliary Results	126
Exercises	130
Chapter 10. Estimation of Regression in Global Norms	131
§10.1. Regressogram	131
§10.2. Integral L_2 -Norm Risk for the Regressogram	133
§10.3. Estimation in the Sup-Norm	136
§10.4. Projection on Span-Space and Discrete MISE	138
§10.5. Orthogonal Series Regression Estimator	141
Exercises	148
Chapter 11. Estimation by Splines	151
§11.1. In Search of Smooth Approximation	151
§11.2. Standard B -splines	152
§11.3. Shifted B -splines and Power Splines	155
§11.4. Estimation of Regression by Splines	158
§11.5. Proofs of Technical Lemmas	161
Exercises	166
Chapter 12. Asymptotic Optimality in Global Norms	167
§12.1. Lower Bound in the Sup-Norm	167
§12.2. Bound in L_2 -Norm. Assouad's Lemma	171
§12.3. General Lower Bound	174
§12.4. Examples and Extensions	177
Exercises	182
Part 3. Estimation in Nonparametric Models	
Chapter 13. Estimation of Functionals	185
§13.1. Linear Integral Functionals	185
§13.2. Non-Linear Functionals	188

Exercises	191
Chapter 14. Dimension and Structure in Nonparametric Regression	193
§14.1. Multiple Regression Model	193
§14.2. Additive regression	196
§14.3. Single-Index Model	199
§14.4. Proofs of Technical Results	206
Exercises	209
Chapter 15. Adaptive Estimation	211
§15.1. Adaptive Rate at a Point. Lower Bound	211
§15.2. Adaptive Estimator in the Sup-Norm	215
§15.3. Adaptation in the Sequence Space	218
§15.4. Proofs of Lemmas	223
Exercises	225
Chapter 16. Testing of Nonparametric Hypotheses	227
§16.1. Basic Definitions	227
§16.2. Separation Rate in the Sup-Norm	229
§16.3. Sequence Space. Separation Rate in the L_2 -Norm	231
Exercises	237
Bibliography	239
Index of Notation	241
Index	243