

# Contents

<b>ANHA Series Preface</b>	<b>v</b>
<b>Preface</b>	<b>xiii</b>
<b>Prologue</b>	<b>xvii</b>
<b>1 Mathematical Background</b>	<b>1</b>
1.1 $\mathbb{R}^n$ and $\mathbb{C}^n$ . . . . .	1
1.2 Abstract vector spaces . . . . .	6
1.3 Finite-dimensional vector spaces . . . . .	9
1.4 Topology in $\mathbb{R}^n$ . . . . .	10
1.5 Supremum and infimum . . . . .	11
1.6 Continuity of functions on $\mathbb{R}$ . . . . .	15
1.7 Integration and summation . . . . .	18
1.8 Some special functions . . . . .	20
1.9 A useful technique: proof by induction . . . . .	22
1.10 Exercises . . . . .	23
<b>2 Normed Vector Spaces</b>	<b>29</b>
2.1 Normed vector spaces . . . . .	29
2.2 Topology in normed vector spaces . . . . .	33
2.3 Approximation in normed vector spaces . . . . .	35
2.4 Linear operators on normed spaces . . . . .	37
2.5 Series in normed vector spaces . . . . .	40
2.6 Exercises . . . . .	42
	<b>ix</b>

<b>3</b>	<b>Banach Spaces</b>	<b>47</b>
3.1	Banach spaces . . . . .	47
3.2	The Banach spaces $\ell^1(\mathbb{N})$ and $\ell^p(\mathbb{N})$ . . . . .	50
3.3	Linear operators on Banach spaces . . . . .	54
3.4	Exercises . . . . .	56
<b>4</b>	<b>Hilbert Spaces</b>	<b>61</b>
4.1	Inner product spaces . . . . .	62
4.2	The Hilbert space $\ell^2(\mathbb{N})$ . . . . .	65
4.3	Orthogonality and direct sum decomposition . . . . .	66
4.4	Functionals on Hilbert spaces . . . . .	69
4.5	Linear operators on Hilbert spaces . . . . .	71
4.6	Bessel sequences in Hilbert spaces . . . . .	75
4.7	Orthonormal bases . . . . .	79
4.8	Frames in Hilbert spaces . . . . .	84
4.9	Exercises . . . . .	85
<b>5</b>	<b>The <math>L^p</math>-spaces</b>	<b>93</b>
5.1	Vector spaces consisting of continuous functions . . . . .	94
5.2	The vector space $L^1(\mathbb{R})$ . . . . .	98
5.3	Integration in $L^1(\mathbb{R})$ . . . . .	103
5.4	The spaces $L^p(\mathbb{R})$ . . . . .	109
5.5	The spaces $L^p(a, b)$ . . . . .	110
5.6	Exercises . . . . .	111
<b>6</b>	<b>The Hilbert Space <math>L^2</math></b>	<b>117</b>
6.1	The Hilbert space $L^2(\mathbb{R})$ . . . . .	117
6.2	Linear operators on $L^2(\mathbb{R})$ . . . . .	120
6.3	The space $L^2(a, b)$ . . . . .	124
6.4	Fourier series revisited . . . . .	126
6.5	Exercises . . . . .	130
<b>7</b>	<b>The Fourier Transform</b>	<b>135</b>
7.1	The Fourier transform on $L^1(\mathbb{R})$ . . . . .	135
7.2	The Fourier transform on $L^2(\mathbb{R})$ . . . . .	142
7.3	Convolution . . . . .	145
7.4	The sampling theorem . . . . .	149
7.5	The discrete Fourier transform . . . . .	154
7.6	Exercises . . . . .	156
<b>8</b>	<b>An Introduction to Wavelet Analysis</b>	<b>159</b>
8.1	Wavelets . . . . .	160
8.2	Multiresolution analysis . . . . .	162
8.3	Vanishing moments and the Daubechies' wavelets . . . . .	168
8.4	Wavelets and signal processing . . . . .	174
8.5	Exercises . . . . .	176

<b>9 A Closer Look at Multiresolution Analysis</b>	<b>181</b>
9.1 Basic properties of multiresolution analysis . . . . .	181
9.2 The spaces $V_j$ and $W_j$ . . . . .	185
9.3 Proof of Theorem 8.2.7 . . . . .	196
9.4 Proof of Theorem 8.2.11 . . . . .	197
9.5 Exercises . . . . .	200
<b>10 B-splines</b>	<b>203</b>
10.1 The B-splines $N_m$ . . . . .	204
10.2 The centered B-splines $B_m$ . . . . .	208
10.3 B-splines and wavelet expansions . . . . .	209
10.4 Frames generated by B-splines . . . . .	210
10.5 Exercises . . . . .	212
<b>11 Special Functions</b>	<b>215</b>
11.1 Regular Sturm–Liouville problems . . . . .	216
11.2 Legendre polynomials . . . . .	222
11.3 Laguerre polynomials . . . . .	228
11.4 Hermite polynomials . . . . .	230
11.5 Exercises . . . . .	232
<b>Appendix A</b>	<b>239</b>
A.1 Proof of Weierstrass’ theorem, Theorem 2.3.4 . . . . .	239
A.2 Proof of Theorem 7.1.7 . . . . .	243
A.3 Proof of Theorem 10.1.5 . . . . .	246
A.4 Proof of Theorem 11.2.2 . . . . .	249
<b>Appendix B</b>	<b>253</b>
B.1 List of vector spaces . . . . .	253
B.2 List of special polynomials . . . . .	255
<b>List of Symbols</b>	<b>257</b>
<b>References</b>	<b>259</b>
<b>Index</b>	<b>261</b>