

Contents

Preface	ix
Chapter 1. Overview	1
1.1. What is a combinatorial design?	1
1.2. What is Algebraic Design Theory?	1
1.3. What is in this book?	2
Chapter 2. Many Kinds of Pairwise Combinatorial Designs	7
2.1. Orthogonality sets	7
2.2. Symmetric balanced incomplete block designs	9
2.3. Hadamard matrices	11
2.4. Weighing matrices	12
2.5. Balanced weighing matrices	13
2.6. Orthogonal designs	14
2.7. Complex Hadamard matrices	16
2.8. Complex generalized Hadamard matrices	18
2.9. Complex generalized weighing matrices	19
2.10. Generalized Hadamard matrices over groups	19
2.11. Balanced generalized weighing matrices	22
2.12. Generalized weighing matrices	23
2.13. Summary	24
Chapter 3. A Primer for Algebraic Design Theory	27
3.1. Groups	27
3.2. Monoids	34
3.3. Group actions	35
3.4. Rings	39
3.5. Matrices	41
3.6. Linear and related groups	42
3.7. Representations	44
Chapter 4. Orthogonality	49
4.1. How many rows can be pairwise Λ -orthogonal?	49
4.2. Non-trivial orthogonality sets	50
4.3. A big picture	51
4.4. Equivalence	54
4.5. Matrices, arrays, and designs	60
Chapter 5. Modeling Λ -Equivalence	63
5.1. A first look at the automorphism group	63
5.2. Ambient rings with a model for Λ -equivalence	65

5.3. Ambient rings for the familiar orthogonality sets	68
Chapter 6. The Grammian	71
6.1. Orthogonality as a Grammian property	71
6.2. Non-degeneracy	72
6.3. Gram completions and composition of orthogonality sets	73
6.4. The Gram Property and Λ -equivalence	74
Chapter 7. Transposability	77
7.1. The main problems	77
7.2. A functional approach to self-duality	78
7.3. Conjugate equivalence operations	80
7.4. A matrix algebra approach to transposability and self-duality	80
7.5. A different kind of transposable orthogonality set	82
Chapter 8. New Designs from Old	85
8.1. Composition	85
8.2. Transference	93
Chapter 9. Automorphism Groups	99
9.1. Automorphism groups of pairwise combinatorial designs	99
9.2. A class of generalized Hadamard matrices	100
9.3. A bound on the size of the automorphism group	103
9.4. Permutation automorphism groups	105
9.5. Automorphism groups of orthogonal designs	106
9.6. Expanded designs	108
9.7. Computing automorphism groups	112
9.8. The associated design	114
9.9. Associated designs and group divisible designs	116
9.10. An isomorphism for weighing matrices	117
Chapter 10. Group Development and Regular Actions on Arrays	119
10.1. Matrix preliminaries	119
10.2. Group-developed arrays	119
10.3. Regular embeddings	121
10.4. Difference sets and relative difference sets	124
10.5. Group ring equations and associates	127
10.6. Finding all associates of an array	129
10.7. An algorithm for solving Problems 10.2.3 and 10.2.4	131
10.8. Composition via associates	132
Chapter 11. Origins of Cocyclic Development	135
11.1. First derivation	135
11.2. Second derivation	140
11.3. Cocycles for cyclic groups	142
Chapter 12. Group Extensions and Cocycles	145
12.1. Central extensions	145
12.2. Cocycles for product groups	150
12.3. Polycyclic presentations	151
12.4. Cocycles from collection in polycyclic groups	153

12.5. Monomial representations and cocycles	157
Chapter 13. Cocyclic Pairwise Combinatorial Designs	161
13.1. The main definitions	161
13.2. Ambient rings with a central group	162
13.3. Some big problems	164
13.4. Central extensions of a design	164
13.5. Approaches to cocyclic designs	165
Chapter 14. Centrally Regular Actions	167
14.1. Cocyclic forms	167
14.2. A lesser expanded design	167
14.3. A pair of lifting homomorphisms	168
14.4. The lift	169
14.5. Translation	170
14.6. Centrally regular embeddings	171
14.7. Finding cocyclic forms	173
14.8. All the cocycles of a design	176
Chapter 15. Cocyclic Associates	177
15.1. Definition of cocyclic associates	177
15.2. The group ring equation for cocyclic associates	178
15.3. The familiar designs	180
15.4. Cocyclic designs and relative difference sets	181
15.5. Normal p -complements	182
15.6. Existence conditions for cocyclic Hadamard matrices	183
15.7. Cyclotomic rings and circulant complex Hadamard matrices	185
15.8. Composition of cocyclic associates	190
Chapter 16. Special Classes of Cocyclic Designs	195
16.1. Cocyclic Hadamard matrices	195
16.2. Cocyclic weighing matrices	197
16.3. Cocyclic orthogonal designs	198
16.4. A cocyclic substitution scheme	200
16.5. Cocyclic complex Hadamard matrices	201
Chapter 17. The Paley Matrices	203
17.1. Actions of 2-dimensional linear and semilinear groups	203
17.2. The Paley matrices and their automorphism groups	205
17.3. The regular actions	209
Chapter 18. A Large Family of Cocyclic Hadamard Matrices	215
18.1. On the orders covered	215
18.2. A construction for prime powers congruent to 3 (mod 4)	216
18.3. A construction for prime powers congruent to 1 (mod 4)	218
18.4. Plug-in matrices	220
18.5. Proof of the main theorem and a generalization	221
Chapter 19. Substitution Schemes for Cocyclic Hadamard Matrices	223
19.1. General substitution schemes	224
19.2. Number-theoretic constraints	226

19.3. Further results for group-developed plug-in matrices	227
19.4. Inverting action	228
19.5. Trivial action	230
19.6. Complementary pairs and the Cocyclic Hadamard Conjecture	232
19.7. Existence of group-developed complementary pairs	233
Chapter 20. Calculating Cocyclic Development Rules	239
20.1. Introduction to development tables	239
20.2. Development tables for abelian groups	240
20.3. Development tables revisited	241
20.4. Group cohomology	242
20.5. Constructing a free table	243
20.6. Group homology	244
20.7. Presentations and the Schur multiplier	246
20.8. Constructing a torsion table	249
20.9. Listing the elements of the second cohomology group	253
20.10. Another look at the Cocyclic Hadamard Conjecture	255
Chapter 21. Cocyclic Hadamard Matrices Indexed by Elementary Abelian Groups	257
21.1. Motivation: indexing groups for the Sylvester matrices	257
21.2. The extension problem	258
21.3. Pure Hadamard collection cocycles	261
21.4. Bilinearity and Hadamard cocycles	262
21.5. Solution of the Hadamard cocycle problem	263
Chapter 22. Cocyclic Concordant Systems of Orthogonal Designs	267
22.1. Existence and uniqueness of cocyclic systems of $\text{OD}(n; 1^k)$	267
22.2. A reduction	268
22.3. Solution of the reduced problem	269
22.4. Proof of Theorem 22.1.1	270
22.5. Removing the zeros	271
22.6. Examples	272
Chapter 23. Asymptotic Existence of Cocyclic Hadamard Matrices	279
23.1. Complex sequences with zero aperiodic autocorrelation	279
23.2. Sets of Hermitian and skew-Hermitian circulant matrices	281
23.3. Sets of cocyclic signed permutation matrices	282
23.4. Existence of cocyclic complex Hadamard matrices	283
23.5. Concluding remarks	284
Bibliography	287
Index	295