

# Contents

<i>Foreword by Michael R. Caputo</i>	<i>page xv</i>
<i>Preface</i>	<i>xvii</i>
1 Introduction	1
Duality, Symmetry, and the Euler-Legendre Transformation	4
Duality without Constraints	6
Asymmetric Duality with Constraints	9
Symmetric Dual Nonlinear Programs	10
Appendix 1.1: The Euler-Legendre Transformation	12
References	14
2 Lagrangean Theory	15
Unconstrained Maximization	16
Concave and Convex Functions	17
Constrained Maximization	18
Saddle Point Problem	20
Homogeneous Functions	21
A Symmetric Lagrangean Function	22
Exercises	25
Reference	27
3 Karush-Kuhn-Tucker Theory	28
Concave Nonlinear Programming	32
Alternative Specifications of Nonlinear Problems	38
Interpretation of Karush-Kuhn-Tucker Conditions	40
Equilibrium Problem	42
How to Solve Nonlinear Programming Problems	44
Exercises	45

Appendix 3.1: Constraint Qualification	46
References	48
4 Solving Systems of Linear Equations	49
Product Form of the Inverse	52
Summary of the Pivot Method's Rules	54
A Numerical Example of the Pivot Method	54
The Geometric Meaning of a Solution	59
Exercises	61
Appendix 4.1: Determinants and Minors	63
Appendix 4.2: Solution of a Linear System of Equations	65
5 Asymmetric and Symmetric Quadratic Programming	66
Preliminaries	66
Asymmetric Quadratic Programming	68
The Dual of the Least-Squares Problem	70
Symmetric Quadratic Programming	72
A Special Case of Self-Duality	74
Numerical Example: The Dual of the Least-Squares Problem	75
GAMS Command File: Least-Squares Example	76
Exercises	78
Appendix 5.1: Differentiation of Linear and Quadratic Forms	80
Appendix 5.2: Eigenvalues and Eigenvectors	81
GAMS Command File for Computing the Eigenvalues of an $(n \times n)$ Symmetric Matrix	83
Appendix 5.3: Integrability Conditions	85
References	87
6 Linear Complementarity Problem	88
The Complementary Pivot Algorithm	90
Example of Symmetric Quadratic Programming as an LC Problem	97
Input Data File for the Lemke Computer Program	100
Output File from the Lemke Computer Program	101
Solving the LCP by Quadratic Programming	103
Example of Solution of LCP by Quadratic Programming	105
Solving Bimatrix Games	107
Exercises	107
References	109

7	The Price Taker	110
	Derivation of the Dual LP Problem	114
	General Linear Model of Joint Production	118
	Numerical Example 1: Linear Joint Production	123
	Numerical Example 2: Two Plants, One Market	125
	GAMS Command File: Numerical Example 2	126
	The Primal Simplex Algorithm	129
	Numerical Example 3: The Primal Simplex Algorithm	132
	The Dual Simplex Algorithm	133
	Numerical Example 4: The Dual Simplex Algorithm	135
	Guidelines to Set Up LP Problems	136
	Exercises	138
	References	140
8	The Monopolist	141
	Pure Monopolist	141
	Perfectly Discriminating Monopolist	145
	Discriminating Monopolist	148
	Perfectly Discriminating Monopolist with Multiple Plants	151
	Pure Monopolist with Asymmetric $D$ Matrix	152
	Numerical Example 1: Pure Monopolist with Asymmetric $D$ Matrix	154
	GAMS Command File: Numerical Example 1	155
	Numerical Example 2: Perfectly Discriminating Monopolist with Symmetric $D$ Matrix	157
	Numerical Example 3: Perfectly Discriminating Monopolist with Asymmetric $D$ Matrix: An Equilibrium Problem	158
	GAMS Command File: Numerical Example 3	159
	Numerical Example 4: Discriminating Monopolist with One Physical Plant and Two Markets	161
	Numerical Example 5: Discriminating Monopolist with Two Physical Plants and Two Markets	163
	GAMS Command File: Numerical Example 5	165
	Exercises	168
9	The Monopsonist	172
	Pure Monopsonist	172
	Perfectly Discriminating Monopsonist	175
	Perfectly Discriminating Monopsonist Respecified	177
	Perfectly Discriminating Monopsonist and Monopsonist by SQP	179

Pure Monopolist and Pure Monopsonist by SQP	181
Pure Monopolist and Pure Monopsonist with Asymmetric $D$ and $G$ Matrices	183
Perfectly Discriminating Monopolist and Perfectly Discriminating Monopsonist with Asymmetric $D$ and $G$ Matrices	184
Numerical Example 1: Price Taker and Pure Monopsonist	186
GAMS Command File: Numerical Example 1	187
Numerical Example 2: Pure Monopolist and Pure Monopsonist by SQP with Asymmetric $D$ and $E$ Matrices	191
Numerical Example 3: Price Taker and Perfectly Discriminating Monopsonist	193
GAMS Command File: Numerical Example 3	194
Exercises	198
10 Risk Programming	202
Risky Output Prices	203
Risky Output Prices and Input Supplies	207
Chance-Constrained Interpretation of Risk Programming	209
Risky Technology	213
Generalization	218
Extension of the Primal-Dual Algorithm to Concave Programs	220
Freund's Numerical Example of Risk Programming	222
GAMS Command File: Freund's Example of Risk Programming	226
Exercises	231
References	234
11 Comparative Statics and Parametric Programming	235
Comparative Statics of the Competitive Firm	236
Parametric Programming in LP Models	236
Comparative Statics in QP Models	240
LP Parametric Programming: Variation in Input Quantity $b_1$	241
LP Parametric Programming: Variation in Output Price $c_2$	246
Parametric Quadratic Programming by LCP	251
Exercises	258
References	259
12 General Market Equilibrium	260
Model 1: Final Commodities	260
Model 2: Intermediate and Final Commodities	266

Model 3: Endogenous Income	269
Model 4: Spatial Equilibrium – One Commodity	271
Model 5: Spatial Equilibrium – Many Commodities	276
Numerical Example 1: General Market Equilibrium Final Commodities	277
GAMS Command File: Numerical Example 1	278
Numerical Example 2: General Market Equilibrium Intermediate and Final Commodities	282
GAMS Command File: Numerical Example 2	284
Numerical Example 3: Spatial Equilibrium – One Commodity	288
GAMS Command File: Numerical Example 3	291
Numerical Example 4: Spatial Equilibrium – Many Commodities	295
GAMS Command File: Numerical Example 4	299
Appendix 12.1: Alternative Specification of GME	304
Appendix 12.2: A Detailed Discussion of Spatial Equilibrium	308
Appendix 12.3: Spatial Equilibrium, Many Commodities	312
Exercises	315
References	317
13 Two-Person Zero- and Non-Zero-Sum Games	318
Two-Person Zero-Sum Games	319
Two-Person Non-Zero-Sum Games	325
Algorithm for Solving a Bimatrix Game	328
A Numerical Example of a Bimatrix Game	329
Maximizing Expected Gain	335
Exercises	338
References	339
14 Positive Mathematical Programming	340
PMP with More Than One Observation	348
Empirical Implementation of PMP	353
Recovering Revenue and Cost Functions	354
Symmetric Positive Equilibrium Problem – SPEP	357
Dynamic Positive Equilibrium Problem – DPEP	361
Numerical Example 1: Dynamic Positive Equilibrium Problem	369
GAMS Command File for Numerical Example 1	376
Revisiting the Three Phases of the Traditional PMP	397
Numerical Example 2: Arfini-Donati PMP Specification	400

	GAMS Command File for Numerical Example 2	403
	Appendix 14.1: Cholesky Factorization	410
	References	410
15	Multiple Optimal Solutions	412
	MOS in Linear Programming	412
	Example of Primal MOS in LP	413
	Dealing with MOS	415
	MOS in QP Models	418
	Determining the Number of Solutions	420
	Example 1: Kaneko's Necessity but not Sufficiency	422
	Example 2: Kaneko's Necessity and Sufficiency	424
	Computing All Basic Feasible and Complementary Solutions	426
	Example 3: LP Problem Revisited	428
	Example 4: QP Problem (15.11) Revisited	433
	Exercises	437
	References	438
16	Lemke Complementary Pivot Algorithm User Manual	439
	I. Purpose	439
	II. Method	442
	III. Limitations	445
	IV. Environment Requirements	445
	V. Input	445
	VI. Output	448
	VII. How to Use the Lemke Program	449
	Example of SQP, Problem 1	450
	Example of an Equilibrium Problem (with Asymmetric <i>D</i> and <i>E</i> Matrices), Problem 5	453
	Example of a General LC Problem, Problem 6	456
	Example of a Bimatrix Game, Problem 7	458
	A Bimatrix Game with Negative Payoffs, Problem 7	462
	References	466
17	Lemke Fortran 77 Program	467
	ReadMe	467
	COMMON.FOR Subroutine	469
	initia.f Subroutine	473
	initia7.f Subroutine	474
	lemke.f Main	478

matprnt.f Subroutine	483
matrd.f Subroutine	483
matrix.f Subroutine	484
newbas.f Subroutine	500
newbas7.f Subroutine	501
opnfil.f Subroutine	502
pinput.f Subroutine	503
pinput6.f Subroutine	505
pivot.f Subroutine	507
pprint.f Subroutine	508
pprint5.f Subroutine	514
pprint6.f Subroutine	519
pprint7.f Subroutine	522
scalam.f Subroutine	527
sort.f Subroutine	527
symval.f Subroutine	529
vecprnt.f Subroutine	537
<i>Index</i>	539