Contents

Part A. Commutative Algebra 1
Introduction 1

1 Rings and Modules 7
1.1 Rings and Ideals 9
1.2 Local Rings and Localization of Rings 18
1.3 Radicals 26
1.4 Modules 31
1.5 Finiteness Conditions and the Snake Lemma 38

2 The Theory of Noetherian Rings 55
2.1 Primary Decomposition of Ideals 57
2.2 Artinian Rings and Modules 66
2.3 The Artin–Rees Lemma 71
2.4 Krull Dimension 74

3 Integral Extensions 83
3.1 Integral Dependence 85
3.2 Noether Normalization and Hilbert's Nullstellensatz 91
3.3 The Cohen–Seidenberg Theorems 96

4 Extension of Coefficients and Descent 103
4.1 Tensor Products 107
4.2 Flat Modules 113
4.3 Extension of Coefficients 123
4.4 Faithfully Flat Descent of Module Properties 131
4.5 Categories and Functors 138
4.6 Faithfully Flat Descent of Modules and their Morphisms 143

5 Homological Methods: Ext and Tor 157
5.1 Complexes, Homology, and Cohomology 159
5.2 The Tor Modules 172
5.3 Injective Resolutions 181
5.4 The Ext Modules 187

Part B. Algebraic Geometry 193
Introduction 193

6 Affine Schemes and Basic Constructions 201
6.1 The Spectrum of a Ring 203
6.2 Functorial Properties of Spectra 212
6.3 Presheaves and Sheaves 216
6.4 Inductive and Projective Limits 222
6.5 Morphisms of Sheaves and Sheafification 232
6.6 Construction of Affine Schemes 241
6.7 The Affine n-Space 255
6.8 Quasi-Coherent Modules 257
6.9 Direct and Inverse Images of Module Sheaves 266

7 Techniques of Global Schemes 277
7.1 Construction of Schemes by Gluing 282
7.2 Fiber Products 294
7.3 Subschemes and Immersions 304
7.4 Separated Schemes 312
7.5 Noetherian Schemes and their Dimension 318
7.6 Čech Cohomology 322
7.7 Grothendieck Cohomology 330

8 Étale and Smooth Morphisms 341
8.1 Differential Forms 343
8.2 Sheaves of Differential Forms 356
8.3 Morphisms of Finite Type and of Finite Presentation 360
8.4 Unramified Morphisms 365
8.5 Smooth Morphisms 374

9 Projective Schemes and Proper Morphisms 399
9.1 Homogeneous Prime Spectra as Schemes 403
9.2 Invertible Sheaves and Serre Twists 418
9.3 Divisors 431
9.4 Global Sections of Invertible Sheaves 446
9.5 Proper Morphisms 462
9.6 Abelian Varieties are Projective 477

Literature 485

Glossary of Notations 487

Index 495