

Contents

Preface	v
1 Introduction	1
1.1 Why random fields?	1
1.2 Some examples	3
1.3 Fundamental concepts	8
1.3.1 Random functions in a broad sense	9
1.3.2 Gaussian random vectors	13
1.3.3 Gaussian random functions	14
1.3.4 Random fields	16
1.3.5 Stochastic measures and integrals	17
1.3.6 Integral representation of random functions	19
1.3.7 Random trajectories	21
1.3.8 Stochastic differential, Ito integrals	22
1.3.9 Brownian motion	22
1.3.10 Multidimensional diffusion and Fokker–Planck equation	25
1.3.11 Central limit theorem and convergence of a Poisson process to a Gaussian process	26
2 Stochastic simulation of vector Gaussian random fields	29
2.1 Introduction	29
2.2 Discrete expansions related to the spectral representations of Gaussian random fields	30
2.2.1 Spectral representations	30
2.2.2 Series expansions	31
2.2.3 Expansion with an even complex orthonormal system	31
2.2.4 Expansion with a real orthonormal system.	32
2.2.5 Complex valued orthogonal expansions	33
2.3 Wavelet expansions	33
2.3.1 Fourier wavelet expansions	34

2.3.2	Wavelet expansion	35
2.3.3	Moving averages	36
2.4	Randomized spectral models	37
2.4.1	Randomized spectral models defined through stochastic integrals	37
2.4.2	Stratified RSM for homogeneous random fields	39
2.5	Fourier wavelet models	39
2.5.1	Meyer wavelet functions	40
2.5.2	Evaluation of the coefficients $\mathcal{F}_m^{(\phi)}$ and $\mathcal{F}_m^{(\psi)}$	40
2.5.3	Cut-off parameters	42
2.5.4	Choice of parameters	43
2.6	Fourier wavelet models of homogeneous random fields based on randomization of plane wave decomposition	47
2.6.1	Plane wave decomposition of homogeneous random fields	47
2.6.2	Decomposition with fixed nodes	50
2.6.3	Decomposition with randomly distributed nodes	52
2.6.4	Some examples	54
2.6.5	Flow in a porous media in the first order approximation	56
2.6.6	Fourier wavelet models of Gaussian random fields	57
2.7	Comparison of Fourier wavelet and randomized spectral models	58
2.7.1	Some technical details of RSM	58
2.7.2	Some technical details of FWM	60
2.7.3	Ensemble averaging	62
2.7.4	Space averaging	62
2.8	Conclusions	63
2.9	Appendices	65
2.9.1	Appendix A. Positive definiteness of the matrix \mathcal{B}	65
2.9.2	Appendix B. Proof of Proposition 2.1	65
3	Stochastic Lagrangian models of turbulent flows: Relative dispersion of a pair of fluid particles	70
3.1	Introduction	70
3.2	Criticism of 2-particle models	73

3.3	The quasi-1-dimensional Lagrangian model of relative dispersion	77
3.3.1	Quasi-1-dimensional analog of formula (2.14a)	78
3.3.2	Models with a finite-order consistency	80
3.3.3	Explicit form of the model (3.26, 3.27)	83
3.3.4	Example	88
3.4	A 3-dimensional model of relative dispersion	90
3.5	Lagrangian models consistent with the Eulerian statistics	92
3.5.1	Diffusion approximation	92
3.5.2	Relation to the well-mixed condition	94
3.5.3	A choice of the coefficients a_i and b_{ij}	95
3.6	Conclusions	97
4	A new Lagrangian model of 2-particle relative turbulent dispersion	98
4.1	Introduction	98
4.2	An examination of Durbin's nonlinear model	98
4.3	Mathematical formulation of a new model	100
4.4	A qualitative analysis of the problem (4.14) for symmetric $\xi(\tau)$	102
4.4.1	Analysis of the problem (4.14) in the deterministic case	102
4.4.2	Analysis of the problem (4.14) for stochastic $\xi(\tau)$	103
4.5	Qualitative analysis of the problem (4.14) in the general case	108
5	The combined Eulerian–Lagrangian model	113
5.1	Introduction	113
5.2	2-particle models	117
5.2.1	Eulerian stochastic models of high-Reynolds-number pseudoturbulence	117
5.3	A new 2-particle Eulerian–Lagrangian stochastic model	120
5.3.1	Formulation of 2-particle Eulerian–Lagrangian model	120
5.3.2	Models for the p. d. f. of the Eulerian relative velocity	123
5.4	Appendix	125
6	Stochastic Lagrangian models for 2-particle relative dispersion in high-Reynolds-number turbulence	129
6.1	Introduction	129

6.2	Preliminaries	130
6.3	A closure of the quasi-1-dimensional model of relative dispersion	131
6.4	Choice of the model (6.1) for isotropic turbulence	132
6.5	The model of relative dispersion of two particles in a locally isotropic turbulence	135
6.5.1	Specification of the model	135
6.5.2	Numerical analysis of the Q1D-model (6.30)	137
6.6	Model of the relative dispersion in intermittent locally isotropic turbulence	139
6.7	Conclusions	141
7	Stochastic Lagrangian models for 2-particle motion in turbulent flows.	
	Numerical results	142
7.1	Introduction	142
7.2	Classical pseudoturbulence model	143
7.2.1	Randomized model of classical pseudoturbulence	143
7.2.2	Mean square separation of two particles in classical pseudoturbulence	146
7.3	Calculations by the combined Eulerian–Lagrangian stochastic model	149
7.3.1	Mean square separation of two particles	149
7.3.2	Thomson’s “two-to-one” reduction principle	152
7.3.3	Concentration fluctuations	154
7.4	Technical remarks	156
7.5	Conclusion	158
8	The 1-particle stochastic Lagrangian model for turbulent dispersion in horizontally homogeneous turbulence	159
8.1	Introduction	159
8.2	Choice of the coefficients in the Ito equation	162
8.3	2D stochastic model with Gaussian p. d. f.	164
8.4	Numerical experiments	167
9	Direct and adjoint Monte Carlo for the footprint problem	171
9.1	Introduction	171

9.2	Formulation of the problem	172
9.3	Stochastic Lagrangian algorithm	173
9.3.1	Direct Monte Carlo algorithm	174
9.3.2	Adjoint algorithm	176
9.4	Impenetrable boundary	178
9.5	Reacting species	180
9.6	Numerical simulations	183
9.7	Conclusion	187
9.8	Appendices	188
9.8.1	Appendix A. Flux representation	188
9.8.2	Appendix B. Probabilistic representation	188
9.8.3	Appendix C. Forward and backward trajectory estimators	189
10	Lagrangian stochastic models for turbulent dispersion in an atmospheric boundary layer	193
10.1	Introduction	193
10.2	Neutrally stratified boundary layer	197
10.2.1	General case of Eulerian p. d. f.	197
10.2.2	Gaussian p. d. f.	200
10.3	Comparison with other models and measurements	201
10.3.1	Comparison with measurements in an ideally-neutral surface layer (INSL)	201
10.3.2	Comparison with the wind tunnel experiment by Raupach and Legg (1983)	204
10.4	Convective case	207
10.5	Boundary conditions	211
10.6	Conclusion	212
10.7	Appendices	213
10.7.1	Appendix A. Derivation of the coefficients in the Gaussian case	213
10.7.2	Appendix B. Relation to other models	215

11 Analysis of the relative dispersion of two particles by Lagrangian stochastic models and DNS methods	218
11.1 Introduction	218
11.2 Basic assumptions	220
11.2.1 Markov assumption	221
11.2.2 Consistency with the second Kolmogorov similarity hypothesis	221
11.2.3 Thomson's well-mixed condition	222
11.3 Well-mixed Lagrangian stochastic models	222
11.3.1 Quadratic-form models	223
11.3.2 Quasi-1-dimensional models	224
11.3.3 3-dimensional extension of Q1D models	225
11.4 Stochastic Lagrangian models based on the moments approximation method	226
11.4.1 Moments approximation conditions	226
11.4.2 Realizability of LS models based on the moments approximation method	227
11.5 Comparison of different models of relative dispersion for the inertial subrange of a fully developed turbulence	229
11.5.1 Q1D quadratic-form model of Borgas and Yeung	229
11.5.2 Comparison of different models in the inertial subrange	231
11.6 Comparison of different Q1D models of relative dispersion for modestly large Reynolds number turbulence ($Re_\lambda \simeq 240$)	232
11.6.1 Parametrization of Eulerian statistics	232
11.6.2 Bi-Gaussian p. d. f.	234
11.6.3 Q1D quadratic-form model	236
12 Evaluation of mean concentration and fluxes in turbulent flows by Lagrangian stochastic models	238
12.1 Introduction	238
12.2 Formulation of the problem	239
12.3 Monte Carlo estimators for the mean concentration and fluxes	243
12.3.1 Forward estimator	244

12.3.2	Modified forward estimators in case of horizontally homogeneous turbulence	245
12.3.3	Backward estimator	250
12.4	Application to the footprint problem	251
12.5	Conclusion	253
12.6	Appendices	253
12.6.1	Appendix A. Representation of concentration in Lagrangian description	253
12.6.2	Appendix B. Relation between forward and backward transition density functions	255
12.6.3	Appendix C. Derivation of the relation between the forward and backward densities	255
13	Stochastic Lagrangian footprint calculations over a surface with an abrupt change of roughness height	258
13.1	Introduction	258
13.2	The governing equations	259
13.2.1	Evaluation of footprint functions	260
13.3	Results	263
13.3.1	Footprint functions of concentration and flux	263
13.4	Discussion and conclusions	276
13.5	Appendices	277
13.5.1	Appendix A. Dimensionless mean-flow equations	277
13.5.2	Appendix B. Lagrangian stochastic trajectory model	278
14	Stochastic flow simulation in 3D porous media	280
14.1	Introduction	280
14.2	Formulation of the problem	283
14.3	Direct numerical simulation method: DSM-SOR	284
14.4	Randomized spectral model (RSM)	286
14.5	Testing the simulation procedure	288
14.6	Evaluation of Eulerian and Lagrangian statistical characteristics by the DNS-SOR method	292
14.6.1	Eulerian statistical characteristics	292

14.6.2	Lagrangian statistical characteristics	294
14.7	Conclusions and discussion	298
15	A Lagrangian stochastic model for the transport in statistically homogeneous porous media	300
15.1	Introduction	300
15.2	Direct simulation method	301
15.2.1	Random flow model	301
15.2.2	Numerical simulation	303
15.2.3	Evaluation of Eulerian characteristics	306
15.2.4	Evaluation of Lagrangian characteristics	310
15.3	Construction of the Langevin-type model	314
15.3.1	Introduction	314
15.3.2	Langevin model for an isotropic porous medium	316
15.3.3	Expressions of the drift terms	319
15.4	Numerical results and comparison against the DSM	321
15.5	Conclusions	321
16	Coagulation of aerosol particles in intermittent turbulent flows	326
16.1	Introduction	326
16.2	Analysis of the fluctuations in the size spectrum	329
16.3	Models of the energy dissipation rate	332
16.3.1	The model by Pope and Chen (<i>P&Ch</i>)	332
16.3.2	The model by Borgas and Sawford (<i>B&S</i>)	334
16.4	Monte Carlo simulation for the Smoluchowski equation in a stochastic coagulation regime	335
16.4.1	The total number of clusters and the mean cluster size	337
16.4.2	The functions $N_3(t)$ and $N_{10}(t)$	339
16.4.3	The size spectrum N_l for different time instances	340
16.4.4	Comparative analysis for two different models of the energy dissipation rate	341
16.5	The case of a coagulation coefficient with no dependence on the cluster size	342
16.6	Simulation of coagulation processes in turbulent coagulation regime	343

16.7 Conclusion	345
16.8 Appendix. Derivation of the coagulation coefficient	346
17 Stokes flows under random boundary velocity excitations	349
17.1 Introduction	349
17.2 Exterior Stokes problem	352
17.2.1 Poisson formula in polar coordinates	353
17.3 K-L expansion of velocity	356
17.3.1 White noise excitations	356
17.3.2 General case of homogeneous excitations	361
17.4 Correlation function of the pressure	366
17.4.1 White noise excitations	366
17.4.2 Homogeneous random boundary excitations	368
17.4.3 Vorticity and stress tensor	368
17.5 Interior Stokes problem	372
17.6 Numerical results	374
Bibliography	381
Index	397