

# Contents

<b>1 General Theory of Primes .....</b>	1
1.1 Prime Places of Associative Rings.....	1
1.2 Primes in Algebras.....	8
1.3 Value Functions and Prime Filtration .....	19
1.4 Dubrovin Valuations .....	33
1.5 Ideal Theory of Dubrovin Valuation Rings .....	42
1.6 Dubrovin Valuation Rings of $Q$ with Finite Dimension Over Its Center .....	58
1.7 Gauss Extensions .....	70
1.8 Filtrations by Totally Ordered Groups .....	88
1.9 Reductions of Algebras and Filtrations .....	96
1.10 Appendix: Global Dimension and Regularity of Reductions .....	101
<b>2 Maximal Orders and Primes .....</b>	109
2.1 Maximal Orders.....	109
2.2 Krull Orders.....	115
2.3 Ore Extensions Over Krull Orders .....	132
2.4 Non-commutative Valuation Rings in $K(X; \sigma, \delta)$ .....	144
2.5 Arithmetical Pseudovaluations and Divisors .....	153
2.6 The Riemann–Roch Theorem for Central Simple Algebras Over Function Fields of Curves .....	164
<b>3 Extensions of Valuations to Quantized Algebras .....</b>	175
3.1 Extension of Central Valuations .....	175
3.2 Discrete Valuations on the Weyl Skewfield .....	183
3.3 Some Divisor Theory for Weyl Fields Over Function Fields .....	193
3.4 Hopf Valuation Filtration .....	197
<b>Bibliography .....</b>	213
<b>Index .....</b>	217