## **Table of contents**

1	Towards New Teaching in Mathematics	1
	Peter Baptist	
	1.1 What is mathematics? – What a question!	. 1
	1.2 Reconsidering one's own teaching – a guiding concept for Fibonacci teachers	. 4
	1.3 Considerations by Günter M. Ziegler: The impact of the Bayreuth Fibonacci Conference (September 2010) on teaching and learning mathematics	. 5
	1.3.1 What is mathematics? Answers by G. M. Ziegler	5
	1.3.2 Comments on Ziegler's concept	7
	1.4 Learning mathematics as inquiry	. 8

## 2 The Basic Patterns as Key Aspects of Inquiry Pedagogy 13 2.1 The concept of basic patterns 13 Dagmar Raab 13 2.1.1 Basic patterns as an underlying core structure 13 2.1.2 What is special about this overarching concept? 13 2.2 How to work with basic patterns 14 Peter Baptist, Dagmar Raab 14 2.2.1 Developing a problem-based culture 14 2.2.2 Promoting cumulative learning 16 2.2.3 Learning from mistakes 18 2.2.4 Experiencing subject boundaries and interdisciplinary approaches 20

IV

	2.3 Dialogic learning — from an educational concept to daily classroom teaching Peter Gallin	23
	2.3.1 Genesis and theory of dialogic learning	23
	2.3.2 Working with "I – You – We" as a teaching aid	
3	IBME and ICT	35
	3.1 The use of dynamic geometry systems (DGS) and computer algebra systems (CAS) in IBME Pavel Pech	35
	3.1.1 Introduction	
	3.1.2 Verification and proving theorems	35
	3.1.3 Deriving theorems	41
	3.1.4 Locus equations	42
	3.1.5 Conclusions	47
	<b>3.2 IBME and ICT – the experience in Bulgaria</b> Petar Kenderov, Evgenia Sendova, Toni Chehlarova	47
	3.2.1 Digital learning environments in support of the IBME	47
	3.2.2 Basic types of dynamic learning environments	
	3.2.3 Discussion	54
	<b>3.3 Interactive geometry for the web and mobile devices</b> Matthias Ehmann, Michael Gerhäuser, Carsten Miller, Heiko Vogel, Alfred Wassermann	54
	3.3.1 Introduction	54
	3.3.2 JSXGraph	56
	3.3.3 JessieCode	59
	3.3.4 Sketchometry	60
	3.3.5 Conclusion	61

V

nere T

.

4 IBME in Schools: Overview and Examples in International Contexts
4.1 Inquiry-based mathematics education in primary school: Overview and examples from Bavaria/Germany65 <i>Volker Ulm</i>
4.1.1 Heterogeneity in primary school65
4.1.2 Aspects of learning69
4.1.3 Inquiry-based learning70
4.1.4 Learning environments for IBME70
4.1.5 Tasks for IBME71
4.1.6 Teaching methods for IBME72
4.1.7 Example for arithmetic: Windows on the hundreds chart
4.1.8 Example for geometry: Quadruples77
<b>4.2 The current state of IBME in the Czech Republic</b>
4.2.1 IBME from the perspective of the Czech framework educational programme82
4.2.2 Twin centre Budweis and its background
4.2.3 Czech teachers and their experience
4.2.4 Digest of learning environments created within the Fibonacci project84
4.2.5 IBME at Czech vocational schools
4.3 IBME in teaching and learning financial literacy topics – selected teaching methods for financial education94 Roman Hašek, Vladimíra Petrášková
4.3.1 Introduction
4.3.2 A teaching method94
4.3.3 Classification of teaching methods94
4.3.4 Problem-based teaching method9
4.3.5 Situation method100
4.3.6 Project method104

.

.

.

4.4 IBME in primary schools in Bulgaria: Some examples of dynamic scenarios and their implementation in a class setting
4.4.1 Counting rectangles
4.4.2 Explorations with a virtual analogue clock
4.5 IBME in the secondary school: Overview and examples in a Bulgarian context
4.5.1 Overview – the lessons from the first IBME&ICT attempts 25 years ago 114
4.5.2 Best practices in designing dynamic scenarios and implementing them in Bulgarian Fibonacci schools115
<b>4.6 IBME in the secondary school: Examples from Switzerland 125</b> <i>Peter Gallin, Markus Jetzer-Caversaccio</i>
4.6.1 On the equilibrium between offer and use – a practical example from a Swiss upper secondary school: The offer-and-use model within the context of dialogic learning <i>Peter Gallin</i>
4.6.2 Dialogic learning: An example from a classroom situation at lower secondary school level
4.7 Fibonacci in Thuringia/Germany – inquiry-based learning in interdisciplinary lessons141 Jörg Triebel
4.7.1 Spirals
4.7.2 The golden section143
4.7.3 The golden angle143
4.7.4 Fibonacci numbers144

,

5	IBME in Teacher Education1	.51
	<b>5.1 In-service teacher training in the Czech Republic</b>	. 151
	5.2 How to encourage teachers to participate	
	in IBME activities Libuse Samkova	. 151
	5.3 The specifics of the teacher education within the Fibonacci project in Bulgaria Evgenia Sendova, Toni Chehlarova	. 154
	<ul> <li>5.3 The specifics of the teacher education within the Fibonacci project in Bulgaria</li> <li>Evgenia Sendova, Toni Chehlarova</li> <li>5.3.1 Teacher training courses promoting a new role for the teachers</li> </ul>	. <b>154</b> 155
	<ul> <li>5.3 The specifics of the teacher education within the Fibonacci project in Bulgaria</li></ul>	. <b>154</b> 155 155
	<ul> <li>5.3 The specifics of the teacher education within the Fibonacci project in Bulgaria</li></ul>	. <b>154</b> 155 155 157

6	Inquiry Based Mathematics Education (IBME) and Gifted Students 163 Petar Kenderov, Evgenia Sendova
	6.1 Introduction163
	6.2 The High School Students' Institute — a Bulgarian model of mathematics and informatics research at school age164
	6.3 The RSI international summer program and the challenge to describe it by one word168

