Contents

Sections denoted with an asterisk (*) can be either omitted or read independently.

Preia	rerace		
Part	I Fundamentals and Techniques of Complex Function Theory	1	
1	Complex Numbers and Elementary Functions	3	
1.1	Complex Numbers and Their Properties	3	
1.2	Elementary Functions and Stereographic Projections	8	
	1.2.1 Elementary Functions	8	
	1.2.2 Stereographic Projections	15	
1.3	Limits, Continuity, and Complex Differentiation	20	
1.4	Elementary Applications to Ordinary Differential Equations	26	
2	Analytic Functions and Integration	32	
2.1	Analytic Functions	32	
	2.1.1 The Cauchy–Riemann Equations	32	
	2.1.2 Ideal Fluid Flow	40	
2.2	Multivalued Functions	46	
2.3	More Complicated Multivalued Functions and Riemann		
	Surfaces	61	
2.4	Complex Integration	70	
2.5	Cauchy's Theorem		
2.6	Cauchy's Integral Formula, Its $\overline{\partial}$ Generalization and		
	Consequences	91	

viii Contents

	2.6.1	Cauchy's Integral Formula and Its Derivatives	91
	*2.6.2	Liouville, Morera, and Maximum-Modulus	
		Theorems	95
	*2.6.3	Generalized Cauchy Formula and $\overline{\partial}$ Derivatives	98
*2.7	Theor	etical Developments	105
3	Seque	ences, Series, and Singularities of Complex Functions	109
3.1	Defini	itions and Basic Properties of Complex Sequences,	
	Series		109
3.2	Taylo	r Series	114
3.3	Laure	nt Series	127
*3.4	Theor	retical Results for Sequences and Series	137
3.5	Singu	larities of Complex Functions	144
	3.5.1	Analytic Continuation and Natural Barriers	152
*3.6	Infini	te Products and Mittag-Leffler Expansions	158
*3.7	Differ	rential Equations in the Complex Plane: Painlevé	
	Equat	ions	174
*3.8	Comp	outational Methods	196
	*3.8.1	Laurent Series	196
	*3.8.2	Differential Equations	198
4	Resid	lue Calculus and Applications of Contour Integration	206
4.1	Caucl	hy Residue Theorem	206
4.2	Evalu	ation of Certain Definite Integrals	217
4.3	Princ	ipal Value Integrals and Integrals with Branch	
	Point	s	237
	4.3.1	Principal Value Integrals	237
	4.3.2	Integrals with Branch Points	245
4.4	The A	Argument Principle, Rouché's Theorem	259
*4.5		er and Laplace Transforms	267
*4.6	Appli	cations of Transforms to Differential Equations	285
Par	t II A _l	pplications of Complex Function Theory	309
5	Conf	ormal Mappings and Applications	31
5.1		duction	31
5.2		ormal Transformations	312
5.3		cal Points and Inverse Mappings	31
5.4	-	ical Applications	322
*5.5	Theo	retical Considerations - Mapping Theorems	34

Contents	ix

5.6	The Schwarz–Christoffel Transformation		
5.7	Bilinear Transformations		
*5.8	Mappi	ings Involving Circular Arcs	382
5.9	Other	Considerations	400
	5.9.1	Rational Functions of the Second Degree	400
	5.9.2	The Modulus of a Quadrilateral	405
	*5.9.3	Computational Issues	408
6	Asym	ptotic Evaluation of Integrals	411
6.1	Introd	411	
	6.1.1	Fundamental Concepts	412
	6.1.2	Elementary Examples	418
6.2	Laplac	ce Type Integrals	422
	6.2.1	Integration by Parts	423
	6.2.2	Watson's Lemma	426
	6.2.3	Laplace's Method	430
6.3	Fourie	er Type Integrals	439
	6.3.1	Integration by Parts	440
	6.3.2	Analog of Watson's Lemma	441
	6.3.3	The Stationary Phase Method	443
6.4	The M	Method of Steepest Descent	448
	6.4.1	Laplace's Method for Complex Contours	453
6.5	Applio	cations	474
6.6	The Stokes Phenomenon		488
	*6.6.1	Smoothing of Stokes Discontinuities	494
6.7	Related Techniques		500
	*6.7.1	WKB Method	500
	*6.7.2	The Mellin Transform Method	504
7	Riema	ann–Hilbert Problems	514
7.1	Introd	luction	514
7.2	Cauch	ny Type Integrals	518
7.3	Scalar Riemann-Hilbert Problems		527
	7.3.1	Closed Contours	529
	7.3.2	Open Contours	533
	7.3.3	Singular Integral Equations	538
7.4	Applic	cations of Scalar Riemann-Hilbert Problems	546
	7.4.1	Riemann-Hilbert Problems on the Real Axis	558
	7.4.2	The Fourier Transform	566
	7/3	The Padon Transform	567

x Contents

*7.5	Matrix Riemann-Hilbert Problems		579
	7.5.1	The Riemann-Hilbert Problem for Rational	
		Matrices	584
	7.5.2	Inhomogeneous Riemann-Hilbert Problems and	
		Singular Equations	586
	7.5.3	The Riemann-Hilbert Problem for Triangular	
		Matrices	587
	7.5.4	Some Results on Zero Indices	589
7.6	The DBAR Problem		
	7.6.1	Generalized Analytic Functions	601
*7.7	Applications of Matrix Riemann–Hilbert Problems and $\bar{\partial}$		
	Problems		604
App	endix A	Answers to Odd-Numbered Exercises	627
Bibli	iography	,	637
Inde	x		640