

CONTENTS

ONE:	INTRODUCTION	1
1.1	The Linear Programming Problem	1
1.2	Linear Programming Modeling and Examples	7
1.3	Geometric Solution.....	17
1.4	The Requirement Space	22
1.5	Notation	27
	Exercises.....	28
	Notes and References	41
TWO:	LINEAR ALGEBRA, CONVEX ANALYSIS, AND POLYHEDRAL SETS	43
2.1	Vectors.....	43
2.2	Matrices	49
2.3	Simultaneous Linear Equations	59
2.4	Convex Sets and Convex Functions.....	62
2.5	Polyhedral Sets and Polyhedral Cones.....	68
2.6	Extreme Points, Faces, Directions, and Extreme Directions of Polyhedral Sets: Geometric Insights	69
2.7	Representation of Polyhedral Sets	73
	Exercises.....	80
	Notes and References	88
THREE:	THE SIMPLEX METHOD	89
3.1	Extreme Points and Optimality	89
3.2	Basic Feasible Solutions	92
3.3	Key to the Simplex Method	101
3.4	Geometric Motivation of the Simplex Method	102
3.5	Algebra of the Simplex Method.....	106
3.6	Termination: Optimality and Unboundedness	112
3.7	The Simplex Method	118
3.8	The Simplex Method in Tableau Format	123
3.9	Block Pivoting	132
	Exercises.....	133
	Notes and References	147
FOUR:	STARTING SOLUTION AND CONVERGENCE	149
4.1	The Initial Basic Feasible Solution	149
4.2	The Two-Phase Method	152
4.3	The Big- M Method.....	163
4.4	How Big Should Big- M Be?	170
4.5	The Single Artificial Variable Technique	171
4.6	Degeneracy, Cycling, and Stalling.....	173
4.7	Validation of the Two Cycling Prevention Rules	179
	Exercises.....	184
	Notes and References	195
FIVE:	SPECIAL SIMPLEX IMPLEMENTATIONS AND OPTIMALITY CONDITIONS	197
5.1	The Revised Simplex Method.....	197
5.2	The Simplex Method for Bounded Variables	217
5.3	Farkas' Lemma via the Simplex Method	230
5.4	The Karush-Kuhn-Tucker Optimality Conditions	233

	Exercises.....	239
	Notes and References	252
SIX:	DUALITY AND SENSITIVITY ANALYSIS.....	255
6.1	Formulation of the Dual Problem	255
6.2	Primal–Dual Relationships	260
6.3	Economic Interpretation of the Dual.....	265
6.4	The Dual Simplex Method.....	273
6.5	The Primal–Dual Method	281
6.6	Finding an Initial Dual Feasible Solution: The Artificial Constraint Technique	289
6.7	Sensitivity Analysis	290
6.8	Parametric Analysis.....	307
	Exercises.....	315
	Notes and References	331
SEVEN:	THE DECOMPOSITION PRINCIPLE	333
7.1	The Decomposition Principle	334
7.2	Numerical Example	339
7.3	Getting Started.....	347
7.4	The Case of Unbounded Region X	348
7.5	Block Diagonal or Angular Structure	355
7.6	Duality and Relationships with other Decomposition Procedures	364
	Exercises.....	369
	Notes and References	382
EIGHT:	COMPLEXITY OF THE SIMPLEX ALGORITHM AND POLYNOMIAL ALGORITHMS.....	383
8.1	Polynomial Complexity Issues	383
8.2	Computational Complexity of the Simplex Algorithm	387
8.3	Khachian's Ellipsoid Algorithm	391
8.4	Karmarkar's Projective Algorithm.....	392
8.5	Analysis of Karmarkar's Algorithm: Convergence, Complexity, Sliding Objective Method, and Basic Optimal Solutions	409
8.6	Affine Scaling, Primal–Dual Path–Following, and Predictor–Corrector Variants of Interior Point Methods.....	420
	Exercises.....	427
	Notes and References	441
NINE:	MINIMAL–COST NETWORK FLOWS.....	445
9.1	The Minimal–Cost Network Flow Problem.....	445
9.2	Some Basic Definitions and Terminology from Graph Theory	447
9.3	Properties of the A Matrix	451
9.4	Representation of a Nonbasic Vector in Terms of the Basic Vectors.....	457
9.5	The Simplex Method for Network Flow Problems	458
9.6	An Example of the Network Simplex Method.....	467
9.7	Finding an Initial Basic Feasible Solution	467
9.8	Network Flows with Lower and Upper Bounds.....	470
9.9	The Simplex Tableau Associated with a Network Flow Problem	473

9.10	List Structures for Implementing the Network	
9.11	Simplex Algorithm	474
9.12	Degeneracy, Cycling, and Stalling.....	480
	Generalized Network Problems	486
	Exercises.....	489
	Notes and References	502
TEN:	THE TRANSPORTATION AND ASSIGNMENT PROBLEMS.....	505
10.1	Definition of the Transportation Problem.....	505
10.2	Properties of the A Matrix	508
10.3	Representation of a Nonbasic Vector in Terms of the Basic Vectors.....	512
10.4	The Simplex Method for Transportation Problems.....	514
10.5	Illustrative Examples and a Note on Degeneracy	520
10.6	The Simplex Tableau Associated with a Transportation Tableau	527
10.7	The Assignment Problem: (Kuhn's) Hungarian Algorithm	527
10.8	Alternating Basis Algorithm for Assignment Problems.....	536
10.9	A Polynomial Successive Shortest Path Approach for Assignment Problems.....	538
10.10	The Transshipment Problem.....	542
	Exercises.....	542
	Notes and References	553
ELEVEN:	THE OUT-OF-KILTER ALGORITHM.....	555
11.1	The Out-of-Kilter Formulation of a Minimal Cost Network Flow Problem	555
11.2	Strategy of the Out-of-Kilter Algorithm.....	562
11.3	Summary of the Out-of-Kilter Algorithm.....	574
11.4	An Example of the Out-of-Kilter Algorithm	576
11.5	A Labeling Procedure for the Out-of-Kilter Algorithm.....	576
11.6	Insight into Changes in Primal and Dual Function Values	579
11.7	Relaxation Algorithms.....	582
	Exercises.....	584
	Notes and References	594
TWELVE:	MAXIMAL FLOW, SHORTEST PATH, MULTICOMMODITY FLOW, AND NETWORK SYNTHESIS PROBLEMS.....	595
12.1	The Maximal Flow Problem	595
12.2	The Shortest Path Problem	607
12.3	Polynomial Shortest Path Algorithms for Networks Having Arbitrary Costs.....	619
12.4	Multicommodity Flows	623
12.5	Characterization of a Basis for the Multicommodity Minimal-Cost Flow Problem.....	633
12.6	Synthesis of Multiterminal Flow Networks	638
	Exercises.....	647
	Notes and References	662
	665
	715
BIBLIOGRAPHY		
INDEX		