
Contents

Preface	xi
Global References	xiii
Other Books	xv
Useful URLs	xix
Common Notation	xxi

1 Matrix Multiplication 1

1.1	Basic Algorithms and Notation	2
1.2	Structure and Efficiency	14
1.3	Block Matrices and Algorithms	22
1.4	Fast Matrix-Vector Products	33
1.5	Vectorization and Locality	43
1.6	Parallel Matrix Multiplication	49

2 Matrix Analysis 63

2.1	Basic Ideas from Linear Algebra	64
2.2	Vector Norms	68
2.3	Matrix Norms	71
2.4	The Singular Value Decomposition	76
2.5	Subspace Metrics	81
2.6	The Sensitivity of Square Systems	87
2.7	Finite Precision Matrix Computations	93

3 General Linear Systems 105

3.1	Triangular Systems	106
3.2	The LU Factorization	111
3.3	Roundoff Error in Gaussian Elimination	122
3.4	Pivoting	125
3.5	Improving and Estimating Accuracy	137
3.6	Parallel LU	144

4 Special Linear Systems 153

4.1	Diagonal Dominance and Symmetry	154
4.2	Positive Definite Systems	159

4.3	Banded Systems	176
4.4	Symmetric Indefinite Systems	186
4.5	Block Tridiagonal Systems	196
4.6	Vandermonde Systems	203
4.7	Classical Methods for Toeplitz Systems	208
4.8	Circulant and Discrete Poisson Systems	219

5 Orthogonalization and Least Squares 233

5.1	Householder and Givens Transformations	234
5.2	The QR Factorization	246
5.3	The Full-Rank Least Squares Problem	260
5.4	Other Orthogonal Factorizations	274
5.5	The Rank-Deficient Least Squares Problem	288
5.6	Square and Underdetermined Systems	298

6 Modified Least Squares Problems and Methods 303

6.1	Weighting and Regularization	304
6.2	Constrained Least Squares	313
6.3	Total Least Squares	320
6.4	Subspace Computations with the SVD	327
6.5	Updating Matrix Factorizations	334

7 Unsymmetric Eigenvalue Problems 347

7.1	Properties and Decompositions	348
7.2	Perturbation Theory	357
7.3	Power Iterations	365
7.4	The Hessenberg and Real Schur Forms	376
7.5	The Practical QR Algorithm	385
7.6	Invariant Subspace Computations	394
7.7	The Generalized Eigenvalue Problem	405
7.8	Hamiltonian and Product Eigenvalue Problems	420
7.9	Pseudospectra	426

8 Symmetric Eigenvalue Problems 439

8.1	Properties and Decompositions	440
8.2	Power Iterations	450
8.3	The Symmetric QR Algorithm	458
8.4	More Methods for Tridiagonal Problems	467
8.5	Jacobi Methods	476
8.6	Computing the SVD	486
8.7	Generalized Eigenvalue Problems with Symmetry	497

9 Functions of Matrices **513**

- 9.1 Eigenvalue Methods 514
- 9.2 Approximation Methods 522
- 9.3 The Matrix Exponential 530
- 9.4 The Sign, Square Root, and Log of a Matrix 536

10 Large Sparse Eigenvalue Problems **545**

- 10.1 The Symmetric Lanczos Process 546
- 10.2 Lanczos, Quadrature, and Approximation 556
- 10.3 Practical Lanczos Procedures 562
- 10.4 Large Sparse SVD Frameworks 571
- 10.5 Krylov Methods for Unsymmetric Problems 579
- 10.6 Jacobi-Davidson and Related Methods 589

11 Large Sparse Linear System Problems **597**

- 11.1 Direct Methods 598
- 11.2 The Classical Iterations 611
- 11.3 The Conjugate Gradient Method 625
- 11.4 Other Krylov Methods 639
- 11.5 Preconditioning 650
- 11.6 The Multigrid Framework 670

12 Special Topics **681**

- 12.1 Linear Systems with Displacement Structure 681
 - 12.2 Structured-Rank Problems 691
 - 12.3 Kronecker Product Computations 707
 - 12.4 Tensor Unfoldings and Contractions 719
 - 12.5 Tensor Decompositions and Iterations 731
- Index 747