

# Contents

<b>1</b>	<b>Hyperfinite Dirichlet Forms</b> .....	1
1.1	Hyperfinite Quadratic Forms .....	2
1.2	Domain of the Symmetric Part .....	6
1.3	Resolvent of the Symmetric Part .....	17
1.4	Weak Coercive Quadratic Forms .....	26
1.5	Hyperfinite Dirichlet Forms .....	36
1.6	Hyperfinite Representations .....	52
1.7	Weak Coercive Quadratic Forms, Revisited .....	61
<b>2</b>	<b>Potential Theory</b> .....	65
2.1	Exceptional Sets .....	66
2.1.1	Exceptional Sets .....	66
2.1.2	Co-Exceptional Sets .....	71
2.2	Excessive Functions and Equilibrium Potentials .....	73
2.3	Capacity Theory .....	78
2.4	Relation of Exceptionality and Capacity Theory .....	86
2.5	Measures of Hyperfinite Energy Integrals .....	91
2.6	Internal Additive Functionals and Associated Measures .....	102
2.7	Fukushima's Decomposition Theorem .....	107
2.7.1	Decomposition Under the Individual Probability Measures $P_i$ .....	107
2.7.2	Decomposition Under the Whole Measure $P$ .....	118
2.8	Internal Multiplicative Functionals .....	120
2.8.1	Internal multiplicative functionals .....	120
2.8.2	Subordinate Semigroups .....	121
2.8.3	Subprocesses .....	122
2.8.4	Feynman-Kac Formulae .....	123
2.9	Alternative Expression of Hyperfinite Dirichlet Forms .....	124
2.10	Transformations of Symmetric Dirichlet Forms .....	125

<b>3</b>	<b>Standard Representation Theory</b> .....	129
3.1	Standard Parts of Hyperfinite Markov Chains .....	130
3.1.1	Inner Standard Part of Sets .....	131
3.1.2	Strong Markov Processes and Modified Standard Parts .....	136
3.2	Hyperfinite Dirichlet Forms and Markov Processes .....	145
3.2.1	Separation of Points .....	146
3.2.2	Nearstandardly Concentrated Forms .....	151
3.2.3	Quasi-Continuity .....	155
3.2.4	Construction of Strong Markov Processes .....	160
<b>4</b>	<b>Construction of Markov Processes</b> .....	165
4.1	Main Result .....	166
4.2	Hyperfinite Lifts of Quasi-Regular Dirichlet Forms .....	170
4.3	Relation with Capacities .....	177
4.4	Path Regularity of Hyperfinite Markov Chains .....	180
4.5	Quasi-Continuity and Nearstandard Concentration .....	181
4.6	Construction of Strong Markov Processes .....	192
4.7	Necessity for Existence of Dual Tight Markov Processes .....	197
<b>5</b>	<b>Hyperfinite Lévy Processes and Applications</b> .....	199
5.1	Standard Lévy Processes .....	200
5.2	Characterizing Hyperfinite Lévy Processes .....	203
5.3	Hyperfinite Lévy Processes: Standard Parts .....	217
5.4	Hyperfinite Lévy-Khintchine Formula .....	224
5.5	Representation Theorem for Lévy Processes .....	232
5.6	Extensions and Applications .....	239
<b>6</b>	<b>Genericity of Loeb Path Spaces</b> .....	243
6.1	Adapted Probability Logic .....	244
6.2	Universality, Saturation, Homogeneity .....	245
6.3	Hyperfinite Adapted Spaces .....	246
6.4	Probability Logic and Markov Processes .....	247
	<b>Appendix</b> .....	249
A.1	General Topology .....	249
A.2	Structure of ${}^*\mathbb{R}$ .....	250
A.3	Internal Sets and Saturation .....	251
A.4	Loeb Measure .....	252
A.5	Linear Spaces .....	253
	<b>Historical Notes</b> .....	255
	<b>References</b> .....	263
	<b>Notation Index</b> .....	281
	<b>Index</b> .....	283