
Contents

Chapter dependencies	<i>page</i>	x
Preface		xi
1 Introduction to probability		1
1.1 Sample spaces, outcomes, and events		6
1.2 Review of set notation		8
1.3 Probability models		17
1.4 Axioms and properties of probability		22
1.5 Conditional probability		26
1.6 Independence		30
1.7 Combinatorics and probability		34
Notes		43
Problems		48
Exam preparation		62
2 Introduction to discrete random variables		63
2.1 Probabilities involving random variables		63
2.2 Discrete random variables		66
2.3 Multiple random variables		70
2.4 Expectation		80
Notes		96
Problems		99
Exam preparation		106
3 More about discrete random variables		108
3.1 Probability generating functions		108
3.2 The binomial random variable		111
3.3 The weak law of large numbers		115
3.4 Conditional probability		117
3.5 Conditional expectation		127
Notes		130
Problems		132
Exam preparation		137
4 Continuous random variables		138
4.1 Densities and probabilities		138
4.2 Expectation of a single random variable		149
4.3 Transform methods		156
4.4 Expectation of multiple random variables		162
4.5 Probability bounds		164
Notes		167
Problems		170
Exam preparation		183
5 Cumulative distribution functions and their applications		184
5.1 Continuous random variables		185
5.2 Discrete random variables		194
5.3 Mixed random variables		197
5.4 Functions of random variables and their cdfs		200
5.5 Properties of cdfs		205
5.6 The central limit theorem		207
5.7 Reliability		215

Notes	219
Problems	222
Exam preparation	238
6 Statistics	240
6.1 Parameter estimators and their properties	240
6.2 Histograms	244
6.3 Confidence intervals for the mean – known variance	250
6.4 Confidence intervals for the mean – unknown variance	253
6.5 Confidence intervals for Gaussian data	256
6.6 Hypothesis tests for the mean	262
6.7 Regression and curve fitting	267
6.8 Monte Carlo estimation	271
Notes	273
Problems	276
Exam preparation	285
7 Bivariate random variables	287
7.1 Joint and marginal probabilities	287
7.2 Jointly continuous random variables	295
7.3 Conditional probability and expectation	302
7.4 The bivariate normal	309
7.5 Extension to three or more random variables	314
Notes	317
Problems	319
Exam preparation	328
8 Introduction to random vectors	330
8.1 Review of matrix operations	330
8.2 Random vectors and random matrices	333
8.3 Transformations of random vectors	340
8.4 Linear estimation of random vectors (Wiener filters)	344
8.5 Estimation of covariance matrices	348
8.6 Nonlinear estimation of random vectors	350
Notes	354
Problems	354
Exam preparation	360
9 Gaussian random vectors	362
9.1 Introduction	362
9.2 Definition of the multivariate Gaussian	363
9.3 Characteristic function	365
9.4 Density function	367
9.5 Conditional expectation and conditional probability	369
9.6 Complex random variables and vectors	371
Notes	373
Problems	375
Exam preparation	382
10 Introduction to random processes	383
10.1 Definition and examples	383
10.2 Characterization of random processes	388
10.3 Strict-sense and wide-sense stationary processes	393
10.4 WSS processes through LTI systems	401
10.5 Power spectral densities for WSS processes	403
10.6 Characterization of correlation functions	410
10.7 The matched filter	412
10.8 The Wiener filter	417

10.9	The Wiener–Khinchin theorem	421
10.10	Mean-square ergodic theorem for WSS processes	423
10.11	Power spectral densities for non-WSS processes	425
	Notes	427
	Problems	429
	Exam preparation	440
11	Advanced concepts in random processes	443
11.1	The Poisson process	443
11.2	Renewal processes	452
11.3	The Wiener process	453
11.4	Specification of random processes	459
	Notes	466
	Problems	466
	Exam preparation	475
12	Introduction to Markov chains	476
12.1	Preliminary results	476
12.2	Discrete-time Markov chains	477
12.3	Recurrent and transient states	488
12.4	Limiting n -step transition probabilities	496
12.5	Continuous-time Markov chains	502
	Notes	507
	Problems	509
	Exam preparation	515
13	Mean convergence and applications	517
13.1	Convergence in mean of order p	518
13.2	Normed vector spaces of random variables	522
13.3	The Karhunen–Loève expansion	527
13.4	The Wiener integral (again)	532
13.5	Projections, orthogonality principle, projection theorem	534
13.6	Conditional expectation and probability	537
13.7	The spectral representation	545
	Notes	549
	Problems	550
	Exam preparation	562
14	Other modes of convergence	564
14.1	Convergence in probability	564
14.2	Convergence in distribution	566
14.3	Almost-sure convergence	572
	Notes	579
	Problems	580
	Exam preparation	589
15	Self similarity and long-range dependence	591
15.1	Self similarity in continuous time	591
15.2	Self similarity in discrete time	595
15.3	Asymptotic second-order self similarity	601
15.4	Long-range dependence	604
15.5	ARMA processes	606
15.6	ARIMA processes	608
	Problems	610
	Exam preparation	613
	Bibliography	615
	Index	618