

## Part I From the Uncertainty Relation to Many-Body Systems

<b>1 Quantum Mechanics of Point Particles . . . . .</b>	<b>3</b>
1.1 Limitations of Classical Physics . . . . .	4
1.2 Heisenberg's Uncertainty Relation for Position and Momentum . . . . .	17
1.2.1 Uncertainties of Observables . . . . .	18
1.2.2 Quantum Mechanical Uncertainties of Canonically Conjugate Variables . . . . .	21
1.2.3 Examples for Heisenberg's Uncertainty Relation . . . . .	24
1.3 The Particle-Wave Dualism . . . . .	26
1.3.1 The Wave Function and its Interpretation . . . . .	28
1.3.2 A First Link to Classical Mechanics . . . . .	31
1.3.3 Gaussian Wave Packet . . . . .	32
1.3.4 Electron in External Electromagnetic Fields . . . . .	35
1.4 Schrödinger Equation and Born's Interpretation of the Wave Function . . . . .	39
1.5 Expectation Values and Observables . . . . .	44
1.5.1 Observables as Self-Adjoint Operators on $L^2(\mathbb{R}^3)$ . . . . .	46
1.5.2 Ehrenfest's Theorem . . . . .	49
1.6 A Discrete Spectrum: Harmonic Oscillator in one Dimension . . . . .	51
1.7 Orthogonal Polynomials in One Real Variable . . . . .	63
1.8 Observables and Expectation Values . . . . .	70
1.8.1 Observables with Nondegenerate Spectrum . . . . .	70
1.8.2 An Example: Coherent States . . . . .	75
1.8.3 Observables with Degenerate, Discrete Spectrum . . . . .	78
1.8.4 Observables with Purely Continuous Spectrum . . . . .	83
1.9 Central Forces and the Schrödinger Equation . . . . .	88
1.9.1 The Orbital Angular Momentum: Eigenvalues and Eigenfunctions . . . . .	88
1.9.2 Radial Momentum and Kinetic Energy . . . . .	98
1.9.3 Force Free Motion with Sharp Angular Momentum . . . . .	100

---

1.9.4	The Spherical Oscillator . . . . .	107
1.9.5	Mixed Spectrum: The Hydrogen Atom . . . . .	113
<b>2</b>	<b>Scattering of Particles by Potentials . . . . .</b>	<b>123</b>
2.1	Macroscopic and Microscopic Scales . . . . .	123
2.2	Scattering on a Central Potential . . . . .	125
2.3	Partial Wave Analysis . . . . .	129
2.3.1	How to Calculate Scattering Phases . . . . .	134
2.3.2	Potentials with Infinite Range: Coulomb Potential . . . . .	138
2.4	Born Series and Born Approximation . . . . .	141
2.4.1	First Born Approximation . . . . .	143
2.4.2	Form Factors in Elastic Scattering . . . . .	145
2.5	*Analytical Properties of Partial Wave Amplitudes . . . . .	149
2.5.1	Jost Functions . . . . .	150
2.5.2	Dynamic and Kinematic Cuts . . . . .	151
2.5.3	Partial Wave Amplitudes as Analytic Functions . . . . .	154
2.5.4	Resonances . . . . .	155
2.5.5	Scattering Length and Effective Range . . . . .	157
2.6	Inelastic Scattering and Partial Wave Analysis . . . . .	160
<b>3</b>	<b>The Principles of Quantum Theory . . . . .</b>	<b>165</b>
3.1	Representation Theory . . . . .	165
3.1.1	Dirac's Bracket Notation . . . . .	168
3.1.2	Transformations Relating Different Representations . . . . .	171
3.2	The Concept of Hilbert Space . . . . .	174
3.2.1	Definition of Hilbert Spaces . . . . .	175
3.2.2	Subspaces of Hilbert Spaces . . . . .	180
3.2.3	Dual Space of a Hilbert Space and Dirac's Notation . . . . .	182
3.3	Linear Operators on Hilbert Spaces . . . . .	183
3.3.1	Self-Adjoint Operators . . . . .	184
3.3.2	Projection Operators . . . . .	188
3.3.3	Spectral Theory of Observables . . . . .	189
3.3.4	Unitary Operators . . . . .	193
3.3.5	Time Evolution of Quantum Systems . . . . .	195
3.4	Quantum States . . . . .	196
3.4.1	Preparation of States . . . . .	197
3.4.2	Statistical Operator and Density Matrix . . . . .	200
3.4.3	Dependence of a State on Its History . . . . .	202
3.4.4	Examples for Preparation of States . . . . .	205
3.5	A First Summary . . . . .	206
3.6	Schrödinger and Heisenberg Pictures . . . . .	208

---

<b>4 Space-Time Symmetries in Quantum Physics . . . . .</b>	<b>211</b>
4.1 The Rotation Group (Part 1) . . . . .	211
4.1.1 Generators of the Rotation Group . . . . .	212
4.1.2 Representations of the Rotation Group . . . . .	214
4.1.3 The Rotation Matrices $D^{(j)}$ . . . . .	220
4.1.4 Examples and Some Formulae for D-Matrices . . . . .	223
4.1.5 Spin and Magnetic Moment of Particles with $j = 1/2$ . . . . .	224
4.1.6 Clebsch-Gordan Series and Coupling of Angular Momenta . . . . .	227
4.1.7 Spin and Orbital Wave Functions . . . . .	230
4.1.8 Pure and Mixed States for Spin 1/2. . . . .	231
4.2 Space Reflection and Time Reversal in Quantum Mechanics . . . . .	233
4.2.1 Space Reflection and Parity . . . . .	233
4.2.2 Reversal of Motion and of Time. . . . .	236
4.2.3 Concluding Remarks on T and $\pi$ . . . . .	239
4.3 Symmetry and Antisymmetry of Identical Particles . . . . .	242
4.3.1 Two Distinct Particles in Interaction . . . . .	242
4.3.2 Identical Particles with the Example $N = 2$ . . . . .	245
4.3.3 Extension to $N$ Identical Particles . . . . .	249
4.3.4 Connection Between Spin and Statistics. . . . .	250
<b>5 Applications of Quantum Mechanics . . . . .</b>	<b>255</b>
5.1 Correlated States and Quantum Information . . . . .	255
5.1.1 Nonlocalities, Entanglement, and Correlations . . . . .	256
5.1.2 Entanglement, More General Considerations . . . . .	262
5.1.3 Classical and Quantum Bits . . . . .	265
5.2 Stationary Perturbation Theory . . . . .	269
5.2.1 Perturbation of a Nondegenerate Energy Spectrum . . . . .	269
5.2.2 Perturbation of a Spectrum with Degeneracy . . . . .	273
5.2.3 An Example: Stark Effect . . . . .	275
5.2.4 Two More Examples: Two-State System, Zeeman-Effect of Hyperfine Structure in Muonium. . . . .	277
5.3 Time Dependent Perturbation Theory and Transition Probabilities . . . . .	285
5.3.1 Perturbative Expansion of Time Dependent Wave Function . . . . .	287
5.3.2 First Order and Fermi's Golden Rule . . . . .	288
5.4 Stationary States of N Identical Fermions. . . . .	291
5.4.1 Self Consistency and Hartree's Method . . . . .	292
5.4.2 The Method of Second Quantization . . . . .	293
5.4.3 The Hartree-Fock Equations . . . . .	296
5.4.4 Hartree-Fock Equations and Residual Interactions. . . . .	299

5.4.5	Particle and Hole States, Normal Product and Wick's Theorem . . . . .	301
5.4.6	Application to the Hartree-Fock Ground State . . . . .	304
<b>Part II From Symmetries in Quantum Physics to Electroweak Interactions</b>		
<b>6</b>	<b>Symmetries and Symmetry Groups in Quantum Physics . . . . .</b>	<b>311</b>
6.1	Action of Symmetries and Wigner's Theorem . . . . .	312
6.1.1	Coherent Subspaces of Hilbert Space . . . . .	313
6.1.2	Wigner's Theorem . . . . .	316
6.2	The Rotation Group (Part 2) . . . . .	319
6.2.1	Relationship between $SU(2)$ and $SO(3)$ . . . . .	320
6.2.2	The Irreducible Unitary Representations of $SU(2)$ . . . . .	323
6.2.3	Addition of Angular Momenta and Clebsch-Gordan Coefficients . . . . .	335
6.2.4	Calculating Clebsch-Gordan Coefficients; the $3j$ -Symbols . . . . .	339
6.2.5	Tensor Operators and Wigner-Eckart Theorem . . . . .	343
6.2.6	*Intertwiner, $6j$ - and $9j$ -Symbols . . . . .	349
6.2.7	Reduced Matrix Elements in Coupled States . . . . .	356
6.2.8	Remarks on Compact Lie Groups and Internal Symmetries . . . . .	359
6.3	Lorentz- and Poincaré Groups . . . . .	364
6.3.1	The Generators of the Lorentz and Poincaré Groups . . . . .	365
6.3.2	Energy-Momentum, Mass and Spin . . . . .	370
6.3.3	Physical Representations of the Poincaré Group . . . . .	372
6.3.4	Massive Single-Particle States and Poincaré Group . . . . .	377
<b>7</b>	<b>Quantized Fields and Their Interpretation . . . . .</b>	<b>383</b>
7.1	The Klein-Gordon Field . . . . .	384
7.1.1	The Covariant Normalization . . . . .	388
7.1.2	A Comment on Physical Units . . . . .	389
7.1.3	Solutions of the Klein-Gordon Equation for Fixed Four-Momentum . . . . .	392
7.1.4	Quantization of the Real Klein-Gordon Field . . . . .	395
7.1.5	Normal Modes, Creation and Annihilation Operators . . . . .	397
7.1.6	Commutator for Different Times, Propagator . . . . .	404
7.2	The Complex Klein-Gordon Field . . . . .	409
7.3	The Quantized Maxwell Field . . . . .	415
7.3.1	Maxwell's Theory in the Lagrange Formalism . . . . .	416
7.3.2	Canonical Momenta, Hamilton- and Momentum Densities . . . . .	419

7.3.3	Lorenz- and Transversal Gauges . . . . .	420
7.3.4	Quantization of the Maxwell Field . . . . .	423
7.3.5	Energy, Momentum, and Spin of Photons . . . . .	426
7.3.6	Helicity and Orbital Angular Momentum of Photons. . . . .	427
7.4	Interaction of the Quantum Maxwell Field with Matter . . . . .	431
7.4.1	Many-Photon States and Matrix Elements . . . . .	432
7.4.2	Absorption and Emission of Single Photons . . . . .	433
7.4.3	Rayleigh- and Thomson Scattering . . . . .	439
7.5	Covariant Quantization of the Maxwell Field . . . . .	445
7.5.1	Gauge Fixing and Quantization. . . . .	446
7.5.2	Normal Modes and One-Photon States. . . . .	448
7.5.3	Lorenz Condition, Energy and Momentum of the Radiation Field . . . . .	450
7.6	*The State Space of Quantum Electrodynamics. . . . .	452
7.6.1	*Field Operators and Maxwell's Equations . . . . .	453
7.6.2	*The Method of Gupta and Bleuler. . . . .	456
7.7	Path Integrals and Quantization. . . . .	460
7.7.1	The Action in Classical Mechanics . . . . .	460
7.7.2	The Action in Quantum Mechanics. . . . .	462
7.7.3	Classical and Quantum Paths . . . . .	465
7.8	Path Integral for Field Theories. . . . .	466
7.8.1	The Functional Derivative . . . . .	467
7.8.2	Functional Power Series and Taylor Series. . . . .	468
7.8.3	Generating Functional . . . . .	469
7.8.4	An Example: Propagator of the Scalar Field. . . . .	471
7.8.5	Complex Scalar Field and Path Integrals . . . . .	473
<b>8</b>	<b>Scattering Matrix and Observables in Scattering and Decays . . . . .</b>	<b>477</b>
8.1	Nonrelativistic Scattering Theory in an Operator Formalism. . . . .	477
8.1.1	The Lippmann-Schwinger Equation. . . . .	477
8.1.2	$T$ -Matrix and Scattering Amplitude . . . . .	481
8.2	Covariant Scattering Theory . . . . .	482
8.2.1	Assumptions and Conventions . . . . .	482
8.2.2	S-Matrix and Optical Theorem . . . . .	483
8.2.3	Cross Sections for Two Particles. . . . .	489
8.2.4	Decay Widths of Unstable Particles . . . . .	495
8.3	Comment on the Scattering of Wave Packets . . . . .	500
<b>9</b>	<b>Particles with Spin 1/2 and the Dirac Equation . . . . .</b>	<b>501</b>
9.1	Relationship Between $SL(2, \mathbb{C}) \curvearrowright \mathbb{L}_+^\dagger$ . . . . .	502
9.1.1	Representations with Spin 1/2 . . . . .	505
9.1.2	*Dirac Equation in Momentum Space . . . . .	507
9.1.3	Solutions of the Dirac Equation in Momentum Space . . . . .	515
9.1.4	Dirac Equation in Spacetime and Lagrange Density . . . . .	519

9.2	Quantization of the Dirac Field . . . . .	523
9.2.1	Quantization of Majorana Fields . . . . .	524
9.2.2	Quantization of Dirac Fields . . . . .	527
9.2.3	Electric Charge, Energy, and Momentum . . . . .	531
9.3	Dirac Fields and Interactions . . . . .	533
9.3.1	Spin and Spin Density Matrix . . . . .	533
9.3.2	The Fermion-Antifermion Propagator . . . . .	539
9.3.3	Traces of Products of $\gamma$ -Matrices . . . . .	541
9.3.4	Chiral States and Their Couplings to Spin-1 Particles . . . . .	546
9.4	When is the Dirac Equation a One-Particle Theory? . . . . .	555
9.4.1	Separation of the Dirac Equation in Polar Coordinates . . . . .	556
9.4.2	Hydrogen-like Atoms from the Dirac Equation . . . . .	560
9.5	Path Integrals with Fermionic Fields . . . . .	568
<b>10</b>	<b>Elements of Quantum Electrodynamics and Weak Interactions . . . . .</b>	<b>573</b>
10.1	S-Matrix and Perturbation Series . . . . .	573
10.1.1	Tools of Quantum Electrodynamics with Leptons . . . . .	577
10.1.2	Feynman Rules for Quantum Electrodynamics with Charged Leptons . . . . .	580
10.1.3	Some Processes in Tree Approximation . . . . .	584
10.2	Radiative Corrections, Regularization, and Renormalization . . . . .	601
10.2.1	Self-Energy of Electrons to Order $\mathcal{O}(e^2)$ . . . . .	601
10.2.2	Renormalization of the Fermion Mass . . . . .	606
10.2.3	Scattering on an External Potential . . . . .	609
10.2.4	Vertex Correction and Anomalous Magnetic Moment . . . . .	617
10.2.5	Vacuum Polarization . . . . .	624
10.3	Epilogue: Quantum Electrodynamics in the Framework of Electroweak Interactions . . . . .	639
10.3.1	Weak Interactions with Charged Currents . . . . .	640
10.3.2	Purely Leptonic Processes and Muon Decay . . . . .	643
10.3.3	Two Simple Semi-leptonic Processes . . . . .	649
<b>Appendix</b>	<b>653</b>	
<b>Historical Notes</b>	<b>681</b>	
<b>Exercises, Hints, and Selected Solutions</b>	<b>691</b>	
<b>References</b>	<b>725</b>	
<b>Index</b>	<b>731</b>	
<b>About the Author</b>	<b>741</b>	