

# Contents

<i>Preface</i>	<i>page</i> xiii
<i>Acronyms and Abbreviations</i>	xvii
1 Introduction	1
1.1 Unobserved Components and Filters	2
1.2 Independence, White Noise and Martingale Differences	5
1.2.1 The Law of Iterated Expectations and Optimal Predictions	5
1.2.2 Definitions and Properties	6
1.3 Volatility	7
1.3.1 Stochastic Volatility	8
1.3.2 Generalized Autoregressive Conditional Heteroscedasticity	8
1.3.3 Exponential GARCH	9
1.3.4 Variance, Scale and Outliers	10
1.3.5 Location/Scale Models	10
1.4 Dynamic Conditional Score Models	11
1.5 Distributions and Quantiles	16
1.6 Plan of Book	17
2 Statistical Distributions and Asymptotic Theory	19
2.1 Distributions	19
2.1.1 Student's $t$ Distribution	20
2.1.2 General Error Distribution	22
2.1.3 Beta Distribution	22
2.1.4 Gamma Distribution	24
2.2 Maximum Likelihood	25
2.2.1 Student's $t$ Distribution	26
2.2.2 General Error Distribution	28
2.2.3 Gamma Distribution	29
2.2.4 Consistency and Asymptotic Normality*	29

2.3	Maximum Likelihood Estimation of Dynamic Conditional Score Models	32
2.3.1	An Information Matrix Lemma	32
2.3.2	Information Matrix for the First-Order Model	34
2.3.3	Information Matrix with the $\delta$ Parameterization*	38
2.3.4	Asymptotic Distribution	39
2.3.5	Consistency and Asymptotic Normality*	40
2.3.6	Nonstationarity	45
2.3.7	Several Parameters	46
2.4	Higher Order Models*	48
2.5	Tests	52
2.5.1	Serial Correlation	52
2.5.2	Goodness of Fit of Distributions	54
2.5.3	Residuals	55
2.5.4	Model Fit	56
2.6	Explanatory Variables	56
3	Location	59
3.1	Dynamic Student's $t$ Location Model	60
3.2	Basic Properties	61
3.2.1	Generalization and Reduced Form	63
3.2.2	Moments of the Observations	63
3.2.3	Autocorrelation Function	64
3.3	Maximum Likelihood Estimation	65
3.3.1	Asymptotic Distribution of the Maximum Likelihood Estimator	65
3.3.2	Monte Carlo Experiments	68
3.3.3	Application to U.S. GDP	69
3.4	Parameter Restrictions*	69
3.5	Higher Order Models and the State Space Form*	70
3.5.1	Linear Gaussian Models and the Kalman Filter	70
3.5.2	The DCS Model	72
3.5.3	QARMA Models	74
3.6	Trend and Seasonality	75
3.6.1	Local Level Model	75
3.6.2	Application to Weekly Hours of Employees in U.S. Manufacturing	77
3.6.3	Local Linear Trend	77
3.6.4	Stochastic Seasonal	79
3.6.5	Application to Rail Travel	80
3.6.6	QARIMA and Seasonal QARIMA Models*	82

3.7	Smoothing	83
3.7.1	Weights	84
3.7.2	Smoothing Recursions for Linear State Space Models	86
3.7.3	Smoothing Recursions for DCS Models	87
3.7.4	Conditional Mode Estimation and the Score	87
3.8	Forecasting	89
3.8.1	QARMA Models	89
3.8.2	State Space Form*	90
3.9	Components and Long Memory	91
3.10	General Error Distribution	93
3.11	Skew Distributions	93
3.11.1	How to Skew a Distribution	94
3.11.2	Dynamic Skew- $t$ Location Model	95
4	Scale	97
4.1	Beta- $t$ -EGARCH	99
4.2	Properties of Stationary Beta- $t$ -EGARCH Models	100
4.2.1	Exponential GARCH	100
4.2.2	Moments	101
4.2.3	Autocorrelation Functions of Squares and Powers of Absolute Values	103
4.2.4	Autocorrelations and Kurtosis	104
4.3	Leverage Effects	105
4.4	Gamma-GED-EGARCH	107
4.5	Forecasting	110
4.5.1	Beta- $t$ -EGARCH	111
4.5.2	Gamma-GED-EGARCH	112
4.5.3	Integrated Exponential Models	113
4.5.4	Predictive Distribution	114
4.6	Maximum Likelihood Estimation and Inference	115
4.6.1	Asymptotic Theory for Beta- $t$ -EGARCH	116
4.6.2	Monte Carlo Experiments	118
4.6.3	Gamma-GED-EGARCH	120
4.6.4	Leverage	121
4.7	Beta- $t$ -GARCH	125
4.7.1	Properties of First-Order Model	125
4.7.2	Leverage Effects	127
4.7.3	Link with Beta- $t$ -EGARCH	127
4.7.4	Estimation and Inference	128
4.7.5	Gamma-GED-GARCH	128
4.8	Smoothing	129
4.9	Application to Hang Seng and Dow Jones	129

4.10	Two Component Models	134
4.11	Trends, Seasonals and Explanatory Variables in Volatility Equations	135
4.12	Changing Location	137
	4.12.1 Explanatory Variables	137
	4.12.2 Stochastic Location and Stochastic Scale	139
4.13	Testing for Changing Volatility and Leverage	141
	4.13.1 Portmanteau Test for Changing Volatility	141
	4.13.2 Martingale Difference Test	142
	4.13.3 Leverage	143
	4.13.4 Diagnostics	144
4.14	Skew Distributions	144
4.15	Time-Varying Skewness and Kurtosis*	148
5	Location/Scale Models for Non-negative Variables	149
5.1	General Properties	151
	5.1.1 Heavy Tails	151
	5.1.2 Moments and Autocorrelations	152
	5.1.3 Forecasts	154
	5.1.4 Asymptotic Distribution of Maximum Likelihood Estimators	154
5.2	Generalized Gamma Distribution	155
	5.2.1 Moments	157
	5.2.2 Forecasts	158
	5.2.3 Maximum Likelihood Estimation	159
5.3	Generalized Beta Distribution	160
	5.3.1 Log-Logistic Distribution	162
	5.3.2 Moments, Autocorrelations and Forecasts	163
	5.3.3 Maximum Likelihood Estimation	164
	5.3.4 Burr Distribution	165
	5.3.5 Generalized Pareto Distribution	166
	5.3.6 $F$ Distribution	167
5.4	Log-Normal Distribution	168
5.5	Monte Carlo Experiments	171
5.6	Leverage, Long Memory and Diurnal Variation	172
5.7	Tests and Model Selection	174
5.8	Estimating Volatility from the Range	176
	5.8.1 Application to Paris CAC and Dow Jones	178
	5.8.2 The Range-EGARCH Model	180
5.9	Duration	181
5.10	Realized Volatility	184
5.11	Count Data and Qualitative Observations	184

6	Dynamic Kernel Density Estimation and Time-Varying Quantiles	187
6.1	Kernel Density Estimation for Time Series	188
6.1.1	Filtering and Smoothing	189
6.1.2	Estimation	191
6.1.3	Correcting for Changing Mean and Variance	193
6.1.4	Specification and Diagnostic Checking	193
6.2	Time-Varying Quantiles	193
6.2.1	Kernel-Based Estimation	194
6.2.2	Direct Estimation of Individual Quantiles	194
6.3	Forecasts	198
6.4	Application to NASDAQ Returns	198
6.4.1	Direct Modelling of Returns	198
6.4.2	ARMA-GARCH Residuals	199
6.4.3	Bandwidth and Tails	201
7	Multivariate Models, Correlation and Association	204
7.1	Multivariate Distributions	205
7.1.1	Estimation	206
7.1.2	Regression	206
7.1.3	Dynamic Models	207
7.2	Multivariate Location Models	209
7.2.1	Structural Time Series Models	209
7.2.2	DCS Model for the Multivariate $t$	210
7.2.3	Asymptotic Theory*	211
7.2.4	Regression and Errors in Variables	212
7.3	Dynamic Correlation	213
7.3.1	A Bivariate Gaussian Model	214
7.3.2	Time-Varying Parameters in Regression	218
7.3.3	Multivariate $t$ Distribution	220
7.3.4	Tests of Changing Correlation	220
7.4	Dynamic Multivariate Scale	220
7.5	Dynamic Scale and Association	222
7.6	Copulas	223
7.6.1	Copulas and Quantiles	223
7.6.2	Measures of Association	226
7.6.3	Maximum Likelihood Estimation	228
7.6.4	Dynamic Copulas	228
7.6.5	Tests Against Changing Association	230
8	Conclusions and Further Directions	231
A	Derivation of Formulae in the Information Matrix	233
A.1	Unconditional Mean Parameterization	233

A.2	Parameterization with $\delta$	235
A.3	Leverage	238
B	Autocorrelation Functions	239
B.1	Beta- $t$ -EGARCH	239
B.2	Gamma-GED-EGARCH	240
B.3	Beta- $t$ -GARCH	241
C	GED Information Matrix	242
D	The Order of GARCH Models	243
E	Computer Programs	245
	<i>Bibliography</i>	247
	<i>Author Index</i>	255
	<i>Subject Index</i>	258