
Contents

Preface	xi
1 Introduction	1
1.1 Generalized Kimura Diffusions	3
1.2 Model Problems	5
1.3 Perturbation Theory	9
1.4 Main Results	10
1.5 Applications in Probability Theory	13
1.6 Alternate Approaches	14
1.7 Outline of Text	16
1.8 Notational Conventions	20
I Wright-Fisher Geometry and the Maximum Principle	23
2 Wright-Fisher Geometry	25
2.1 Polyhedra and Manifolds with Corners	25
2.2 Normal Forms and Wright-Fisher Geometry	29
3 Maximum Principles and Uniqueness Theorems	34
3.1 Model Problems	34
3.2 Kimura Diffusion Operators on Manifolds with Corners	35
3.3 Maximum Principles for the Heat Equation	45
II Analysis of Model Problems	49
4 The Model Solution Operators	51
4.1 The Model Problem in 1-dimension	51
4.2 The Model Problem in Higher Dimensions	54
4.3 Holomorphic Extension	59
4.4 First Steps Toward Perturbation Theory	62
5 Degenerate Hölder Spaces	64
5.1 Standard Hölder Spaces	65
5.2 WF-Hölder Spaces in 1-dimension	66

6	Hölder Estimates for the 1-dimensional Model Problems	78
6.1	Kernel Estimates for Degenerate Model Problems	80
6.2	Hölder Estimates for the 1-dimensional Model Problems	89
6.3	Properties of the Resolvent Operator	103
7	Hölder Estimates for Higher Dimensional Corner Models	107
7.1	The Cauchy Problem	109
7.2	The Inhomogeneous Case	122
7.3	The Resolvent Operator	135
8	Hölder Estimates for Euclidean Models	137
8.1	Hölder Estimates for Solutions in the Euclidean Case	137
8.2	1-dimensional Kernel Estimates	139
9	Hölder Estimates for General Models	143
9.1	The Cauchy Problem	145
9.2	The Inhomogeneous Problem	149
9.3	Off-diagonal and Long-time Behavior	166
9.4	The Resolvent Operator	169
III	Analysis of Generalized Kimura Diffusions	179
10	Existence of Solutions	181
10.1	WF-Hölder Spaces on a Manifold with Corners	182
10.2	Overview of the Proof	187
10.3	The Induction Argument	191
10.4	The Boundary Parametrix Construction	194
10.5	Solution of the Homogeneous Problem	205
10.6	Proof of the Doubling Theorem	208
10.7	The Resolvent Operator and C_0 -Semi-group	209
10.8	Higher Order Regularity	211
11	The Resolvent Operator	218
11.1	Construction of the Resolvent	220
11.2	Holomorphic Semi-groups	229
11.3	Diffusions Where All Coefficients Have the Same Leading Homogeneity	230
12	The Semi-group on $\mathcal{C}^0(P)$	235
12.1	The Domain of the Adjoint	237
12.2	The Null-space of \bar{L}^\ominus	240
12.3	Long Time Asymptotics	243
12.4	Irregular Solutions of the Inhomogeneous Equation	247

A Proofs of Estimates for the Degenerate 1-d Model	251
A.1 Basic Kernel Estimates	252
A.2 First Derivative Estimates	272
A.3 Second Derivative Estimates	278
A.4 Off-diagonal and Large- t Behavior	291
Bibliography	301
Index	305