
Contents

Preface	xi
Chapter 1. Introduction	1
§1.1. Basic themes	1
§1.2. Classical and quantum mechanics	3
§1.3. Overview	5
§1.4. Notes	9
Part 1. BASIC THEORY	
Chapter 2. Symplectic geometry and analysis	13
§2.1. Flows	13
§2.2. Symplectic structure on \mathbb{R}^{2n}	14
§2.3. Symplectic mappings	16
§2.4. Hamiltonian vector fields	20
§2.5. Lagrangian submanifolds	23
§2.6. Notes	26
Chapter 3. Fourier transform, stationary phase	27
§3.1. Fourier transform on \mathcal{S}	27
§3.2. Fourier transform on \mathcal{S}'	35
§3.3. Semiclassical Fourier transform	38
§3.4. Stationary phase in one dimension	40

§3.5. Stationary phase in higher dimensions	46
§3.6. Oscillatory integrals	52
§3.7. Notes	54
Chapter 4. Semiclassical quantization	55
§4.1. Definitions	56
§4.2. Quantization formulas	59
§4.3. Composition, asymptotic expansions	65
§4.4. Symbol classes	72
§4.5. Operators on L^2	81
§4.6. Compactness	87
§4.7. Inverses, Gårding inequalities	90
§4.8. Notes	96
Part 2. APPLICATIONS TO PARTIAL DIFFERENTIAL EQUATIONS	
Chapter 5. Semiclassical defect measures	99
§5.1. Construction, examples	99
§5.2. Defect measures and PDE	104
§5.3. Damped wave equation	106
§5.4. Notes	117
Chapter 6. Eigenvalues and eigenfunctions	119
§6.1. The harmonic oscillator	119
§6.2. Symbols and eigenfunctions	124
§6.3. Spectrum and resolvents	129
§6.4. Weyl's Law	132
§6.5. Notes	137
Chapter 7. Estimates for solutions of PDE	139
§7.1. Classically forbidden regions	140
§7.2. Tunneling	143
§7.3. Order of vanishing	148
§7.4. L^∞ estimates for quasimodes	152
§7.5. Schauder estimates	158
§7.6. Notes	167

Part 3. ADVANCED THEORY AND APPLICATIONS

Chapter 8. More on the symbol calculus	171
§8.1. Beals's Theorem	171
§8.2. Real exponentiation of operators	177
§8.3. Generalized Sobolev spaces	182
§8.4. Wavefront sets, essential support, and microlocality	187
§8.5. Notes	196
Chapter 9. Changing variables	197
§9.1. Invariance, half-densities	197
§9.2. Changing symbols	203
§9.3. Invariant symbol classes	206
§9.4. Notes	217
Chapter 10. Fourier integral operators	219
§10.1. Operator dynamics	220
§10.2. An integral representation formula	226
§10.3. Strichartz estimates	235
§10.4. L^p estimates for quasimodes	240
§10.5. Notes	244
Chapter 11. Quantum and classical dynamics	245
§11.1. Egorov's Theorem	245
§11.2. Quantizing symplectic mappings	251
§11.3. Quantizing linear symplectic mappings	257
§11.4. Egorov's Theorem for longer times	264
§11.5. Notes	271
Chapter 12. Normal forms	273
§12.1. Overview	273
§12.2. Normal forms: real symbols	275
§12.3. Propagation of singularities	279
§12.4. Normal forms: complex symbols	282
§12.5. Quasimodes, pseudospectra	286
§12.6. Notes	289

Chapter 13. The FBI transform	291
§13.1. Motivation	291
§13.2. Complex analysis	293
§13.3. FBI transforms and Bergman kernels	302
§13.4. Quantization and Toeplitz operators	311
§13.5. Applications	321
§13.6. Notes	336

Part 4. SEMICLASSICAL ANALYSIS ON MANIFOLDS

Chapter 14. Manifolds	339
§14.1. Definitions, examples	339
§14.2. Pseudodifferential operators on manifolds	345
§14.3. Schrödinger operators on manifolds	354
§14.4. Notes	362
Chapter 15. Quantum ergodicity	365
§15.1. Classical ergodicity	366
§15.2. A weak Egorov Theorem	368
§15.3. Weyl's Law generalized	370
§15.4. Quantum ergodic theorems	372
§15.5. Notes	379

Part 5. APPENDICES

Appendix A. Notation	383
§A.1. Basic notation	383
§A.2. Functions, differentiation	385
§A.3. Operators	387
§A.4. Estimates	388
§A.5. Symbol classes	389
Appendix B. Differential forms	391
§B.1. Definitions	391
§B.2. Push-forwards and pull-backs	394
§B.3. Poincaré's Lemma	396
§B.4. Differential forms on manifolds	397

Appendix C. Functional analysis	399
§C.1. Operator theory	399
§C.2. Spectral theory	403
§C.3. Trace class operators	411
Appendix D. Fredholm theory	415
§D.1. Grushin problems	415
§D.2. Fredholm operators	416
§D.3. Meromorphic continuation	418
Bibliography	421
Index	427