Contents

	Acknowledgements		*page* x
	Introduction		1
		What is this book?	1
		What this book is not	1
		Get started	1
		Short history of geodynamics and numerical geodynamic modelling	4
		A few words about programming and visualisation	7
		How to use this book	8
		Programming exercises and homework	9
1	The continuity equation		11
	1.1	Continuum – what is it?	11
	1.2	Continuity equation	13
	1.3	Eulerian and Lagrangian points – what is the difference?	14
	1.4	Derivation of the Eulerian continuity equation	15
	1.5	Derivation of the Lagrangian continuity equation	18
	1.6	Comparing Eulerian and Lagrangian continuity equations. Advective transport term	20
	1.7	Incompressible continuity equation	23
		Analytical exercise	23
		Programming exercise and homework	24
2	Density and gravity		25
	2.1	Density of rocks and minerals. Equations of state	25
	2.2	Gravity and gravitational potential	30
		Analytical exercise	34
		Programming exercises and homework	35

Gerya, Taras
Introduction to numerical geodynamic modelling
2011

digitalisiert durch:
IDS Basel Bern

3 Numerical solutions of partial differential equations 37
3.1 Finite-difference method 37
3.2 Solving linear equations 43
3.3 Geometrical and global indexing of unknowns 47
Programming exercises and homework 48
4 Stress and strain 51
4.1 Stress 51
4.2 Strain and strain rate 56
Analytical exercise 59
Programming exercise and homework 60
5 The momentum equation 61
5.1 Momentum equation 61
5.2 Newtonian law of viscous friction 64
5.3 Navier–Stokes equation 65
5.4 Poisson equation 68
5.5 Stream function approach 69
Analytical exercise 71
Programming exercise and homework 71
6 Viscous rheology of rocks 73
6.1 Rock rheology 73
6.2 Effective viscosity 74
6.3 Non-Newtonian channel flow 79
Programming exercises and homework 80
7 Numerical solutions of the momentum and continuity equations 83
7.1 Grids 83
7.2 Discretisation of the equations 86
7.3 Conservative finite differences 87
7.4 Boundary conditions 92
7.5 Indexing of unknowns 95
Programming exercises and homework 101
8 The advection equation and marker-in-cell method 105
8.1 Advection equation 105
8.2 Eulerian advection methods 106
8.3 Marker-in-cell techniques 113
Programming exercises and homework 119
9 The heat conservation equation 123
9.1 Fourier's law of heat conduction 123
9.2 Heat conservation equation 124
9.3 Heat generation and consumption 127

9.4 Simplified temperature equations 128
9.5 Heat diffusion timescales 129
Analytical exercises 130
Programming exercises and homework 131
10 Numerical solution of the heat conservation equation 133
10.1 Explicit and implicit formulation of the temperature equation 133
10.2 Conservative finite differences 135
10.3 Advection of temperature with Eulerian methods 140
10.4 Advection of temperature with markers 141
10.5 Thermal boundary conditions 144
Programming exercises and homework 146
11 2D thermomechanical code structure 149
11.1 What do we expect from geodynamic codes? 149
11.2 Thermomechanical code structure 150
11.3 Adding self-gravity and free surface 158
Programming exercise and homework 163
12 Elasticity and plasticity 165
12.1 Why care about elasticity and plasticity? 165
12.2 Elastic rheology 165
12.3 Rotation of elastic stresses 168
12.4 Maxwell visco-elastic rheology 172
12.5 Plastic rheology 173
12.6 Visco-elasto-plastic rheology 175
Analytical exercise 177
Programming exercises and homework 177
13 2D implementation of visco-elasto-plastic rheology 179
13.1 Viscous-like reformulation of visco-elasto-plasticity 179
13.2 Structure of visco-elasto-plastic thermomechanical code 180
13.3 Visco-elasto-plastic iterations 189
Programming exercises and homework 191
14 The multigrid method 193
14.1 Multigrid – what is it? 193
14.2 Solving the Poisson equation with multigrid 200
14.3 Solving Stokes and continuity equations with multigrid 205
Programming exercises and homework 217
15 Programming of 3D problems 221
15.1 Why simply not always 3D? 221

15.2 3D staggered grid and discretisation of momentum, continuity, temperature and Poisson equations 222
15.3 Solving discretised 3D equations 231
Programming exercises and homework 239
16 Numerical benchmarks 241
16.1 Code benchmarking: why should we spend time on it? 241
16.2 Test 1. Rayleigh–Taylor instability benchmark 242
16.3 Test 2. Falling block benchmark 244
16.4 Test 3. Channel flow with a non-Newtonian rheology 246
16.5 Test 4. Non-steady temperature distribution in a Newtonian channel 247
16.6 Test 5. Couette flow with viscous heating 250
16.7 Test 6. Advection of sharp temperature fronts 253
16.8 Test 7. Channel flow with variable thermal conductivity 253
16.9 Test 8. Thermal convection with constant and variable viscosity 255
16.10 Test 9. Stress build-up in a visco-elastic Maxwell body 260
16.11 Test 10. Recovery of the original shape of an elastic slab 261
16.12 Test 11. Numerical sandbox benchmark 263
16.13 Possible further benchmarks 267
Programming exercises and homework 267
17 Design of 2D numerical geodynamic models 269
17.1 Warning message! 269
17.2 What is numerical modelling all about? 269
17.3 Material properties 270
17.4 Visco-elasto-plastic slab bending 271
17.5 Retreating oceanic subduction 276
17.6 Lithospheric extension 279
17.7 Continental collision 282
17.8 Slab breakoff 287
17.9 Intrusion emplacement into the crust 291
17.10 Mantle convection with phase changes 296
17.11 Deformation of self-gravitating planetary body 301
Programming exercise and homework 306
Epilogue: outlook 307
Where are we now? 307
Where to go further? 307
State-of-the-art overview 311
Efficient direct solvers 312
Parallelisation of numerical codes 313

Mesh refinement algorithms 313
Including complex realistic physics in numerical geodynamic models 314
3D visualisation challenges 317
Conceptual warning 318
Conclusion 318
Appendix: MATLAB program examples 319
References 326
Index 340