

# Contents

<b>1</b>	<b>Introduction</b> .....	1
<b>2</b>	<b>Background</b> .....	11
2.1	Real Quadratic Number Fields .....	11
2.2	Fuchsian Groups .....	16
2.3	Congruence Subgroups of $SL_2(\mathcal{O}_D)$ .....	19
2.4	Moduli Spaces .....	22
2.4.1	Flat Surfaces and Veech Groups .....	22
2.4.2	The Moduli Space of Compact Riemann Surfaces .....	24
2.4.3	Strata .....	26
2.4.4	The Moduli Space of Abelian Varieties $\mathcal{A}_g^D$ .....	28
2.5	Hilbert Modular Surfaces .....	32
2.5.1	Special Algebraic Curves on Hilbert Modular Surfaces .....	35
<b>3</b>	<b>Teichmüller Curves</b> .....	39
3.1	Definition and Main Properties .....	39
3.2	Examples in Genus 2 .....	42
3.3	Fixing the Veech Group .....	46
<b>4</b>	<b>Twisted Teichmüller Curves</b> .....	53
<b>5</b>	<b>Stabilizer and Maximality</b> .....	61
5.1	Maximal Fuchsian Groups and the Commensurator .....	62
5.2	Pseudo Parabolic Maximal Groups .....	70
5.3	Stabilizer and Commensurator .....	78
<b>6</b>	<b>Calculations for Twisted Teichmüller Curves</b> .....	85
6.1	The Volume of Diagonal Twisted Teichmüller Curves .....	86
6.1.1	Divisors of Split Prime Numbers .....	90
6.1.2	Inert Prime Numbers .....	93
6.1.3	Divisors of Ramified Prime Numbers .....	97
6.1.4	Prime Ideals with $(\mathfrak{p}, \eta^*) \neq 1$ .....	100

6.2	The Volume of Upper Triangular Twists .....	102
6.2.1	The Relatively Prime Case.....	103
6.2.2	The Non-relatively Prime Case .....	107
6.3	Classification of Twisted Teichmüller Curves .....	108
6.4	An Outlook on Further Calculations.....	114
<b>7</b>	<b>Prym Varieties and Teichmüller Curves</b> .....	<b>121</b>
7.1	Prym Varieties .....	121
7.2	Prym Varieties and Teichmüller Curves .....	122
7.3	Hilbert Modular Embeddings .....	124
<b>8</b>	<b>Lyapunov Exponents</b> .....	<b>127</b>
8.1	Basic Theory .....	127
8.2	The Geodesic Flow .....	129
8.3	The Teichmüller Flow .....	130
8.3.1	The Teichmüller Flow on $\Omega\mathcal{M}_g$ .....	130
8.3.2	The Teichmüller Flow on Teichmüller Curves.....	131
8.4	The Kontsevich-Zorich Cocycle and Its Lyapunov Exponents .....	132
<b>9</b>	<b>Kobayashi Curves Revisited</b> .....	<b>135</b>
9.1	An Arithmetic Approach .....	135
9.2	The General Case.....	139
<b>A</b>	<b>Appendix</b> .....	<b>145</b>
A.1	Proof of Theorem 2.30 .....	145
A.2	Elements of Non-cocompact Cofinite Fuchsian Groups.....	149
A.3	Checking Commensurability .....	152
<b>B</b>	<b>Tables</b> .....	<b>155</b>
	<b>References</b> .....	<b>159</b>
	<b>Index</b> .....	<b>165</b>