

Contents

Preface	xv
Acknowledgements	xvii
1 Multivariate Linear Time Series	1
1.1 Introduction, 1	
1.2 Some Basic Concepts, 5	
1.2.1 Stationarity, 5	
1.2.2 Linearity, 6	
1.2.3 Invertibility, 7	
1.3 Cross-Covariance and Correlation Matrices, 8	
1.4 Sample CCM, 9	
1.5 Testing Zero Cross-Correlations, 12	
1.6 Forecasting, 16	
1.7 Model Representations, 18	
1.8 Outline of the Book, 22	
1.9 Software, 23	
Exercises, 23	
2 Stationary Vector Autoregressive Time Series	27
2.1 Introduction, 27	
2.2 VAR(1) Models, 28	
2.2.1 Model Structure and Granger Causality, 28	
2.2.2 Relation to Transfer Function Model, 30	
2.2.3 Stationarity Condition, 31	
2.2.4 Invertibility, 32	
2.2.5 Moment Equations, 32	
2.2.6 Implied Models for the Components, 35	
2.2.7 Moving-Average Representation, 36	

- 2.3 VAR(2) Models, 37
 - 2.3.1 Stationarity Condition, 37
 - 2.3.2 Moment Equations, 38
 - 2.3.3 Implied Marginal Component Models, 40
 - 2.3.4 Moving-Average Representation, 40
- 2.4 VAR(p) Models, 41
 - 2.4.1 A VAR(1) Representation, 41
 - 2.4.2 Stationarity Condition, 42
 - 2.4.3 Moment Equations, 42
 - 2.4.4 Implied Component Models, 43
 - 2.4.5 Moving-Average Representation, 43
- 2.5 Estimation, 44
 - 2.5.1 Least-Squares Methods, 44
 - 2.5.2 Maximum Likelihood Estimate, 47
 - 2.5.3 Limiting Properties of LS Estimate, 49
 - 2.5.4 Bayesian Estimation, 55
- 2.6 Order Selection, 61
 - 2.6.1 Sequential Likelihood Ratio Tests, 61
 - 2.6.2 Information Criteria, 63
- 2.7 Model Checking, 66
 - 2.7.1 Residual Cross-Correlations, 66
 - 2.7.2 Multivariate Portmanteau Statistics, 71
 - 2.7.3 Model Simplification, 72
- 2.8 Linear Constraints, 80
- 2.9 Forecasting, 82
 - 2.9.1 Forecasts of a Given Model, 82
 - 2.9.2 Forecasts of an Estimated Model, 84
- 2.10 Impulse Response Functions, 89
 - 2.10.1 Orthogonal Innovations, 92
- 2.11 Forecast Error Variance Decomposition, 96
- 2.12 Proofs, 98
- Exercises, 100

3 Vector Autoregressive Moving-Average Time Series

105

- 3.1 Vector MA Models, 106
 - 3.1.1 VMA(1) Model, 106
 - 3.1.2 Properties of VMA(q) Models, 110

- 3.2 Specifying VMA Order, 112
- 3.3 Estimation of VMA Models, 113
 - 3.3.1 Conditional Likelihood Estimation, 113
 - 3.3.2 Exact Likelihood Estimation, 116
 - 3.3.3 Initial Parameter Estimation, 126
- 3.4 Forecasting of VMA Models, 126
- 3.5 VARMA Models, 127
 - 3.5.1 Identifiability, 128
 - 3.5.2 VARMA(1,1) Models, 130
 - 3.5.3 Some Properties of VARMA Models, 133
- 3.6 Implications of VARMA Models, 139
 - 3.6.1 Granger Causality, 139
 - 3.6.2 Impulse Response Functions, 141
- 3.7 Linear Transforms of VARMA Processes, 141
- 3.8 Temporal Aggregation of VARMA Processes, 144
- 3.9 Likelihood Function of a VARMA Model, 146
 - 3.9.1 Conditional Likelihood Function, 146
 - 3.9.2 Exact Likelihood Function, 150
 - 3.9.3 Interpreting the Likelihood Function, 152
 - 3.9.4 Computation of Likelihood Function, 154
- 3.10 Innovations Approach to Exact Likelihood Function, 155
 - 3.10.1 Block Cholesky Decomposition, 157
- 3.11 Asymptotic Distribution of Maximum Likelihood Estimates, 160
 - 3.11.1 Linear Parameter Constraints, 162
- 3.12 Model Checking of Fitted VARMA Models, 163
- 3.13 Forecasting of VARMA Models, 164
 - 3.13.1 Forecasting Updating, 166
- 3.14 Tentative Order Identification, 166
 - 3.14.1 Consistent AR Estimates, 166
 - 3.14.2 Extended Cross-Correlation Matrices, 169
 - 3.14.3 A Summary Two-Way Table, 171
- 3.15 Empirical Analysis of VARMA Models, 176
 - 3.15.1 Personal Income and Expenditures, 176
 - 3.15.2 Housing Starts and Mortgage Rate, 184
- 3.16 Appendix, 192
- Exercises, 194

4 Structural Specification of VARMA Models

- 4.1 The Kronecker Index Approach, 200
 - 4.1.1 A Predictive Interpretation, 205
 - 4.1.2 A VARMA Specification, 207
 - 4.1.3 An Illustrative Example, 208
 - 4.1.4 The Echelon Form, 211
 - 4.1.5 The Example Continued, 212
 - 4.2 The Scalar Component Approach, 212
 - 4.2.1 Scalar Component Models, 213
 - 4.2.2 Model Specification Via Scalar Component Models, 215
 - 4.2.3 Redundant Parameters, 216
 - 4.2.4 VARMA Model Specification, 218
 - 4.2.5 The Transformation Matrix, 218
 - 4.3 Statistics for Order Specification, 220
 - 4.3.1 Reduced Rank Tests, 220
 - 4.4 Finding Kronecker Indices, 222
 - 4.4.1 Application, 224
 - 4.5 Finding Scalar Component Models, 226
 - 4.5.1 Implication of Scalar Component Models, 227
 - 4.5.2 Exchangeable Scalar Component Models, 229
 - 4.5.3 Searching for Scalar Components, 232
 - 4.5.4 Application, 233
 - 4.6 Estimation, 237
 - 4.6.1 Illustration of the Kronecker Index Approach, 238
 - 4.6.2 Illustration of the SCM Approach, 241
 - 4.7 An Example, 245
 - 4.7.1 The SCM Approach, 245
 - 4.7.2 The Kronecker Index Approach, 252
 - 4.7.3 Discussion and Comparison, 257
 - 4.8 Appendix: Canonical Correlation Analysis, 259
- Exercises, 262

5 Unit-Root Nonstationary Processes

- 5.1 Univariate Unit-Root Processes, 266
 - 5.1.1 Motivation, 267
 - 5.1.2 Unit Root with Stationary Innovations, 269
 - 5.1.3 AR(1) Case, 274

- 5.1.4 AR(p) Case, 274
- 5.1.5 MA(1) Case, 276
- 5.1.6 Unit-Root Tests, 276
- 5.1.7 Example, 277
- 5.2 Multivariate Unit-Root Processes, 279
 - 5.2.1 An Alternative Model Representation, 282
 - 5.2.2 Unit-Root VAR Processes, 285
- 5.3 Spurious Regressions, 290
- 5.4 Multivariate Exponential Smoothing, 291
- 5.5 Cointegration, 294
 - 5.5.1 An Example of Cointegration, 295
 - 5.5.2 Some Justifications of Cointegration, 297
- 5.6 An Error-Correction Form, 297
- 5.7 Implications of Cointegrating Vectors, 300
 - 5.7.1 Implications of the Deterministic Term, 300
 - 5.7.2 Implications for Moving-Average Representation, 301
- 5.8 Parameterization of Cointegrating Vectors, 302
- 5.9 Cointegration Tests, 303
 - 5.9.1 The Case of VAR Models, 303
 - 5.9.2 Specification of Deterministic Terms, 304
 - 5.9.3 Review of Likelihood Ratio Tests, 305
 - 5.9.4 Cointegration Tests of VAR Models, 306
 - 5.9.5 An Illustration, 309
 - 5.9.6 Cointegration Tests of VARMA Models, 313
- 5.10 Estimation of Error-Correction Models, 313
 - 5.10.1 VAR Models, 313
 - 5.10.2 Reduced Regression Method, 317
 - 5.10.3 VARMA Models, 318
- 5.11 Applications, 319
- 5.12 Discussion, 326
- 5.13 Appendix, 327
- Exercises, 328

6 Factor Models and Selected Topics

333

- 6.1 Seasonal Models, 333
- 6.2 Principal Component Analysis, 341
- 6.3 Use of Exogenous Variables, 345

- 6.3.1 VARX Models, 346
- 6.3.2 Regression Model, 352
- 6.4 Missing Values, 357
 - 6.4.1 Completely Missing, 358
 - 6.4.2 Partially Missing, 361
- 6.5 Factor Models, 364
 - 6.5.1 Orthogonal Factor Models, 364
 - 6.5.2 Approximate Factor Models, 370
 - 6.5.3 Diffusion Index Models, 371
 - 6.5.4 Dynamic Factor Models, 375
 - 6.5.5 Constrained Factor Models, 376
 - 6.5.6 Asymptotic Principal Component Analysis, 380
- 6.6 Classification and Clustering Analysis, 386
 - 6.6.1 Clustering Analysis, 386
 - 6.6.2 Bayesian Estimation, 387
 - 6.6.3 An MCMC Procedure, 390
- Exercises, 394

7 Multivariate Volatility Models

399

- 7.1 Testing Conditional Heteroscedasticity, 401
 - 7.1.1 Portmanteau Test, 401
 - 7.1.2 Rank-Based Test, 402
 - 7.1.3 Simulation, 403
 - 7.1.4 Application, 405
- 7.2 Estimation of Multivariate Volatility Models, 407
- 7.3 Diagnostic Checks of Volatility Models, 409
 - 7.3.1 Ling and Li Statistics, 409
 - 7.3.2 Tse Statistics, 412
- 7.4 Exponentially Weighted Moving Average, 414
- 7.5 BEKK Models, 417
 - 7.5.1 Discussion, 420
- 7.6 Cholesky Decomposition and Volatility Modeling, 420
 - 7.6.1 Volatility Modeling, 422
 - 7.6.2 Application, 423
- 7.7 Dynamic Conditional Correlation Models, 428
 - 7.7.1 A Procedure for Building DCC Models, 430
 - 7.7.2 An Example, 430
- 7.8 Orthogonal Transformation, 434

- 7.8.1 The Go-GARCH Model, 434
- 7.8.2 Dynamic Orthogonal Components, 438
- 7.8.3 Testing the Existence of DOC, 440
- 7.9 Copula-Based Models, 443
 - 7.9.1 Copulas, 444
 - 7.9.2 Gaussian and t -Copulas, 445
 - 7.9.3 Multivariate Volatility Modeling, 449
- 7.10 Principal Volatility Components, 454
 - 7.10.1 Sample Principal Volatility Components, 458
- Exercises, 461

Appendix A Review of Mathematics and Statistics

465

- A.1 Review of Vectors and Matrices, 465
 - A.1.1 Basic Operations, 466
 - A.1.2 Inverse, Trace, Eigenvalue, and Eigenvector, 467
 - A.1.3 Positive-Definite Matrix, 467
 - A.1.4 Comparison of Two Symmetric Matrices, 469
 - A.1.5 Idempotent Matrix, 469
 - A.1.6 Cholesky Decomposition, 469
 - A.1.7 Partition of a Matrix, 470
 - A.1.8 Vectorization and Kronecker Product, 470
 - A.1.9 Vector and Matrix Differentiation, 472
- A.2 Least-Squares Estimation, 477
- A.3 Multivariate Normal Distributions, 478
- A.4 Multivariate Student- t Distribution, 479
- A.5 Wishart and Inverted Wishart Distributions, 480
- A.6 Vector and Matrix Differentials, 481
 - A.6.1 Scalar Function, 481
 - A.6.2 Vector Function, 484
 - A.6.3 Matrix Function, 485
 - A.6.4 Properties of Matrix Differential, 485
 - A.6.5 Application, 486

Index

489