

Contents

Preface	vii
1 Floquet Theory	1
1.1 Introduction	1
1.2 Preliminaries on ordinary differential systems	1
1.3 Periodic first-order systems	5
1.4 The discriminant and stability	9
1.5 Hill's equation and periodic Dirac systems	12
1.6 Functional properties of Hill's discriminant	15
1.7 The Mathieu equation	19
1.8 Periodic, semi-periodic and twisted boundary-value problems	21
1.9 Appendix: Rofe-Beketov's formula	24
1.10 Chapter notes	26
2 Oscillations	31
2.1 Introduction	31
2.2 The Prüfer transform	32
2.3 The boundary-value problem with separated boundary conditions	35
2.4 The rotation number	40
2.5 Zeros of solutions of Hill's equation	46
2.6 The upper end-points of the stability intervals	48
2.7 A step-function example	52
2.8 Even coefficients	55
2.9 Comparison of eigenvalues	57
2.10 Least eigenvalues	60
2.11 Chapter notes	64
3 Asymptotics	67
3.1 Introduction	67
3.2 Prüfer transformation formulae	67
3.3 The coefficient w	70
3.4 Titchmarsh's asymptotic formula	73

3.5	Differentiable q	76
3.6	Length of the instability intervals	79
3.7	The Mathieu equation	86
3.8	Asymptotic formulae for solutions	91
3.9	Absence of instability intervals	94
3.10	Absence of all but N finite instability intervals	98
3.11	Absence of odd instability intervals	102
3.12	All instability intervals non-vanishing	105
3.13	Chapter notes	108
4	Spectra	113
4.1	Introduction	113
4.2	Regular boundary-value problems	114
4.3	The spectral function for the half-line problem	120
4.4	Self-adjoint half-line operators	130
4.5	The spectrum of the periodic boundary-value problem on the half-line	136
4.6	The spectral matrix for the full-line problem	142
4.7	The spectrum of the full-line periodic problem	148
4.8	Oscillations and spectra	151
4.9	Bounded solutions and the absolutely continuous spectrum	154
4.10	Chapter notes	157
5	Perturbations	161
5.1	Introduction	161
5.2	Spectral bands	162
5.3	Gap eigenvalues	168
5.4	Critical coupling constants	175
5.5	Eigenvalue asymptotics	184
5.6	Chapter notes	193
Bibliography		197
Index		213