

# CONTENTS

PREFACE	v
Some symbols	vi
Chapter 1. SETS	1
11. Introduction	1
12. Sets and their members	3
13. Inclusion	4
14. Set formation	5
15. Special sets	7
16. Basic operations	10
17. Pairs; product sets	14
18. Partitions	17
Chapter 2. MAPPINGS	19
21. The concept of a mapping	19
22. The graph of a mapping	21
23. The range of a mapping; images and pre-images; the partition of a mapping	23
24. Inclusion, identity, and partition mappings	27
25. Composition of mappings; diagrams; restrictions and adjustments	28
26. Mappings from a set to itself	31
Chapter 3. PROPERTIES OF MAPPINGS	33
31. Constants	33
32. Injective, surjective, and bijective mappings	34
33. Inverses and invertibility	37
34. Injectivity, surjectivity, and bijectivity: The induced mappings	41
35. Cancellability	44
36. Factorization	46
Chapter 4. FAMILIES	51
41. The concept of a family	51
42. Special families	53
43. Families of sets	55
44. Products and direct unions	59
45. General associative and distributive laws	64

46. Set-products and set-coproducts . . . . .	67
Chapter 5. RELATIONS . . . . .	71
51. Relations in a set . . . . .	71
52. Images and pre-images . . . . .	73
53. Reversal, composition, and restriction of relations . . . . .	74
54. Relations from set to set; functional relations . . . . .	76
55. Properties of relations . . . . .	78
56. Order . . . . .	80
57. Equivalence relations . . . . .	82
Chapter 6. ORDERED SETS . . . . .	87
61. Basic concepts . . . . .	87
62. Isotone mappings . . . . .	93
63. Products . . . . .	97
64. Properties of ordered sets . . . . .	100
65. Lexicographic products and ordered direct unions . . . . .	102
Chapter 7. COMPLETELY ORDERED SETS . . . . .	105
71. Completely ordered sets . . . . .	105
72. Pre-completely ordered sets . . . . .	109
73. Closure mappings . . . . .	114
74. Galois correspondences . . . . .	118
75. The fixed-point theorem for isotone mappings . . . . .	121
Chapter 8. INDUCTION AND RECURSION . . . . .	123
81. Proof by induction . . . . .	123
82. Recursive definitions . . . . .	125
Chapter 9. THE NATURAL NUMBERS . . . . .	133
91. Principles of counting . . . . .	133
92. Order . . . . .	136
93. General induction and recursive definitions . . . . .	140
94. Iteration . . . . .	143
95. Essential uniqueness of counting systems . . . . .	146
96. Addition and subtraction . . . . .	148
97. Multiplication and division . . . . .	152
98. Divisors and multiples . . . . .	156
Chapter 10. FINITE SETS . . . . .	159
101. Finite sets and their cardinals . . . . .	159
102. Induction . . . . .	163

103. Operations with finite sets . . . . .	165
104. Factorials and binomial coefficients . . . . .	169
105. Orders in finite sets . . . . .	174
106. Finiteness without counting . . . . .	177
 Chapter 11. FINITE SUMS . . . . .	 183
111. Commutative monoids . . . . .	183
112. Finite sums . . . . .	185
113. Sums of families with finite support . . . . .	188
114. Repeated and double sums . . . . .	191
115. Natural multiples . . . . .	194
116. The Inclusion-Exclusion Principle . . . . .	196
117. Sums in monoids of families . . . . .	200
118. Sums without zero . . . . .	205
 Chapter 12. COUNTABLE SETS . . . . .	 209
121. Countable sets . . . . .	209
122. Some uncountable sets . . . . .	216
123. Another characterization of finiteness . . . . .	219
 Chapter 13. SOME ALGEBRAIC STRUCTURES . . . . .	 221
131. Commutative monoids and groups . . . . .	221
132. Commutative rings . . . . .	225
133. Fields . . . . .	229
 Chapter 14. THE REAL NUMBERS: COMPLETE ORDERED FIELDS . . . . .	 231
141. Introduction . . . . .	231
142. Ordered fields . . . . .	233
143. Complete ordered fields . . . . .	238
144. Essential uniqueness of complete ordered fields . . . . .	241
 Chapter 15. THE REAL-NUMBER SYSTEM . . . . .	 247
151. The Real-Number System . . . . .	247
152. The Extended-Real-Number System . . . . .	252
152. Binary digital expansion . . . . .	256
 Chapter 16. THE REAL NUMBERS: EXISTENCE . . . . .	 261
161. Construction of a complete ordered field . . . . .	261
162. Construction of a positivity system . . . . .	266
163. Existence . . . . .	271
 Chapter 17. INFINITE SETS . . . . .	 273
171. Introduction . . . . .	273

172. Maximality principles . . . . . 275  
173. Collections of finitary character . . . . . 277  
174. The Axiom of Choice . . . . . 280  
175. Comparison of sets . . . . . 283  
176. Well-ordered sets . . . . . 290  
177. Completing the proof of equivalence . . . . . 295

INDEXES . . . . . 297  
Index of terms . . . . . 297  
Index of names . . . . . 304  
Index of conditions . . . . . 305  
Index of symbols . . . . . 306