
Contents

Preface	ix
Acknowledgements	xii
Chapter I. Projective modules and vector bundles	1
§1. Free modules, GL_n , and stably free modules	1
§2. Projective modules	8
§3. The Picard group of a commutative ring	20
§4. Topological vector bundles and Chern classes	34
§5. Algebraic vector bundles	49
Chapter II. The Grothendieck group K_0	69
§1. The group completion of a monoid	69
§2. K_0 of a ring	74
§3. $K(X)$, $KO(X)$, and $KU(X)$ of a topological space	89
§4. Lambda and Adams operations	98
§5. K_0 of a symmetric monoidal category	114
§6. K_0 of an abelian category	124
§7. K_0 of an exact category	140
§8. K_0 of schemes and varieties	157
§9. K_0 of a Waldhausen category	172
Appendix. Localizing by calculus of fractions	189
Chapter III. K_1 and K_2 of a ring	197
§1. The Whitehead group K_1 of a ring	197

§2. Relative K_1	212
§3. The Fundamental Theorems for K_1 and K_0	217
§4. Negative K -theory	229
§5. K_2 of a ring	236
§6. K_2 of fields	251
§7. Milnor K -theory of fields	266
Chapter IV. Definitions of higher K -theory	283
§1. The BGL^+ definition for rings	284
§2. K -theory with finite coefficients	304
§3. Geometric realization of a small category	311
§4. Symmetric monoidal categories	326
§5. λ -operations in higher K -theory	341
§6. Quillen's Q -construction for exact categories	347
§7. The " $+ = Q$ " Theorem	358
§8. Waldhausen's wS_* construction	364
§9. The Gillet-Grayson construction	377
§10. Nonconnective spectra in K -theory	381
§11. Karoubi-Villamayor K -theory	385
§12. Homotopy K -theory	394
Chapter V. The Fundamental Theorems of higher K -theory	401
§1. The Additivity Theorem	401
§2. Waldhausen localization and approximation	413
§3. The Resolution Theorems and transfer maps	423
§4. Devissage	439
§5. The Localization Theorem for abelian categories	442
§6. Applications of the Localization Theorem	445
§7. Localization for $K_*(R)$ and $K_*(X)$	462
§8. The Fundamental Theorem for $K_*(R)$ and $K_*(X)$	472
§9. The coniveau spectral sequence of Gersten and Quillen	477
§10. Descent and Mayer-Vietoris properties	486
§11. Chern classes	494
Chapter VI. The higher K -theory of fields	509
§1. K -theory of algebraically closed fields	509
§2. The e -invariant of a field	516

§3. The K -theory of \mathbb{R}	523
§4. Relation to motivic cohomology	527
§5. K_3 of a field	536
§6. Global fields of finite characteristic	552
§7. Local fields	558
§8. Number fields at primes where $cd = 2$	564
§9. Real number fields at the prime 2	568
§10. The K -theory of \mathbb{Z}	579
Bibliography	589
Index of notation	599
Index	605