

Contents

Part I Preliminaries

1	Introduction	3
1.1	Classical Bifurcation Versus Bifurcation Without Parameters	3
1.2	Manifolds of Equilibria	6
1.2.1	Conserved Quantities	6
1.2.2	Equivariances	7
1.2.3	Reversibilities	7
1.2.4	Singular Perturbations	8
1.2.5	Perturbing the Manifold	9
1.2.6	Cosymmetries	9
1.3	Classification of Bifurcation Types	10
1.3.1	Codimension One	10
1.3.2	Codimension Two	10
1.4	Further Cases	12
2	Methods and Concepts	13
2.1	Center Manifolds	13
2.2	Normal Forms	14
2.3	Normal Forms with Manifolds of Equilibria	16
2.4	Genericity	16
2.5	Unfoldings and Codimension	17
2.6	Rescaling and Blow Up	18
3	Cosymmetries	21

Part II Codimension One

4	Transcritical Bifurcation	27
4.1	The Generic Case	27
4.2	Additional Reflection Symmetry	30

5	Poincaré-Andronov-Hopf Bifurcation	35
6	Application: Decoupling in Networks	43
7	Application: Oscillatory Profiles in Systems of Hyperbolic Balance Laws	49

Part III Codimension Two

8	Degenerate Transcritical Bifurcation	57
8.1	Families of Lines of Equilibria: Singular Drift	57
8.2	Families of Lines of Equilibria: Fold	60
8.3	Planes of Equilibria	61
9	Degenerate Poincaré-Andronov-Hopf Bifurcation	67
9.1	Families of Lines of Equilibria: Singular Drift	68
9.2	Families of Lines of Equilibria: Fold	73
9.3	Planes of Equilibria	75
10	Bogdanov-Takens Bifurcation	81
10.1	Normal Form	82
10.2	Integrable Core	86
10.3	Poincaré Flow	87
10.4	Elliptic Integrals and the Riccati Equation	88
10.5	Discussion of the Poincaré Flow	92
10.6	Poincaré Return Map and Bounded Solutions	96
11	Zero-Hopf Bifurcation	103
12	Double-Hopf Bifurcation	109
12.1	Family of Lines of Equilibria	109
12.2	Plane of Equilibria	112
13	Application: Cosmological Models of Bianchi Type, the Tumbling Universe	115
14	Application: Fluid Flow in a Planar Channel, Spatial Dynamics with Reversible Bogdanov-Takens Bifurcation	119
14.1	Fully Symmetric Case	122
14.2	Symmetry-Breaking Perturbations	124

Part IV Beyond Codimension Two

15	Codimension-One Manifolds of Equilibria	131
16	Summary and Outlook	135
16.1	Singularity Theory	135
16.2	Symmetries	136

Contents	ix
16.3 Global Bifurcation.....	136
16.4 Recurrence.....	137
References.....	139