

Correlations, Entanglement and Quantum Phases in Low-Dimensional Frustrated Spin Systems

Dissertation

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