

# Contents

Preface — vi

Notation — xiv

List of Figures — xvi

**1 How positive discrete dynamical systems do arise — 1**

1.1 Non-linear population dynamics in one dimension — 1

Exercises — 6

1.2 The density dependent Leslie model — 7

Exercises — 11

1.3 Non-linear price dynamics in one dimension — 12

Exercises — 13

1.4 The Leontief model with choice of techniques — 14

Exercises — 16

1.5 Opinion dynamics under bounded confidence — 18

Exercises — 19

Bibliography — 19

**2 Concave Perron–Frobenius theory — 21**

2.1 Iteration of normalized concave operators — 21

Exercises — 30

2.2 Indecomposability and primitivity for ray-preserving concave operators — 32

Exercises — 41

2.3 Concave operators which are positively homogeneous — 42

Exercises — 51

2.4 A special case: Linear Perron–Frobenius theory — 53

Exercises — 56

2.5 Applications to difference equations of concave type — 57

Exercises — 61

2.6 Relative stability in the concave Leslie model — 62

Exercises — 67

2.7 Price setting and balanced growth in a concave Leontief model — 68

Exercises — 71

Bibliography — 72

**3 Internal metrics on convex cones — 76**

3.1 Extraction within convex cones — 76

Exercises — 82

<b>3.2</b>	<b>Internal metrics — 84</b>
	<b>Exercises — 89</b>
<b>3.3</b>	<b>Geometrical properties — 90</b>
	<b>Exercises — 100</b>
<b>3.4</b>	<b>Completeness for internal metrics — 101</b>
	<b>Exercises — 114</b>
	<b>Bibliography — 115</b>
<b>4</b>	<b>Contractive dynamics on metric spaces — 118</b>
<b>4.1</b>	<b>Iteration of contractive selfmappings — 118</b>
	<b>Exercises — 122</b>
<b>4.2</b>	<b>Non-autonomous discrete systems — 122</b>
	<b>Exercises — 128</b>
<b>4.3</b>	<b>A local-global stability principle for power-lipschitzian mappings — 129</b>
	<b>Exercises — 132</b>
	<b>Bibliography — 133</b>
<b>5</b>	<b>Ascending dynamics in convex cones of infinite dimension — 135</b>
<b>5.1</b>	<b>Definition and examples of ascending operators — 135</b>
	<b>Exercises — 145</b>
<b>5.2</b>	<b>Relative stability for ascending operators by Hilbert's projective metric — 146</b>
	<b>Exercises — 155</b>
<b>5.3</b>	<b>Absolute stability for weakly ascending operators by the part metric — 156</b>
	<b>Exercises — 164</b>
<b>5.4</b>	<b>Applications to nonlinear difference equations and to nonlinear integral operators — 166</b>
	<b>Exercises — 171</b>
	<b>Bibliography — 173</b>
<b>6</b>	<b>Limit set trichotomy — 176</b>
<b>6.1</b>	<b>Weak and strong forms of limit set trichotomy in Banach spaces — 177</b>
<b>6.2</b>	<b>Differentiability criteria for non-expansiveness and contractivity — 188</b>
<b>6.3</b>	<b>Applications to nonlinear difference equations and cooperative systems of differential equations — 199</b>
	<b>Exercises — 211</b>
	<b>Bibliography — 214</b>

<b>7</b>	<b>Non-autonomous positive systems — 216</b>
7.1	The concepts of path stability, asymptotic proportionality, weak and strong ergodicity — 217
7.2	Path stability and weak ergodicity for ascending operators — 221
7.3	Strong ergodicity for ascending operators — 229
7.4	A nonlinear version of Poincaré's theorem on nonautonomous difference equations — 234
7.5	Price setting in case of technical change — 241
7.6	Populations under bounded and periodic enforcement — 246
	Exercises — 251
	Bibliography — 254
<b>8</b>	<b>Dynamics of interaction: opinions, mean maps, multi-agent coordination, and swarms — 257</b>
8.1	Scrambling matrices — 258
8.2	Consensus formation and opinion dynamics under bounded confidence — 269
8.3	Mean processes, mean structures and the iteration of mean maps — 273
8.4	Infinite products of stochastic matrices: path stability, convergence and a generalized theorem of Wolfowitz — 289
8.5	Multi-agent coordination and opinion dynamics — 300
8.6	Swarm dynamics — 323
	Exercises — 334
	Bibliography — 339
	Index — 345