

CONTENTS

Preface	ix
1 Introduction and mathematical preliminaries	1
1.1 Introduction	1
1.2 Summation notation	2
1.3 Factorial notation $n!$	2
1.4 Combinatorial notation $\binom{n}{r}$	3
1.5 Power notation	5
1.6 Differentiation; the slope of a curve	6
1.7 Maxima and minima	10
*1.8 Functions of more than one variable; maxima and minima	12
1.9 The exponential function e^x	14
1.10 The natural logarithm function $\ln x$	18
1.11 Exercises	23
2 Elementary probability	25
2.1 Introduction; concept of probability	25
2.2 Joint and disjoint events; intersection and union	26
2.3 Conditional probability	30
2.4 Independence of two events	31
2.5 Exercises	32
3 Random variables and their distributions	34
3.1 Discrete random variables and their distributions	34
3.2 Continuous random variables and their distributions	38
3.3 The area under a curve; integration and differentiation	42
3.4 Exercises	50
4 Location and dispersion	52
4.1 Measures of location – mean, median and mode	52
4.2 Dispersion – variance and standard deviation	57
**4.3 Expectations and moments	61
**4.4 Conditional means	63
**4.5 Conditional variances	64
**4.6 Skewness	65
4.7 Exercises	66
5 Statistical distributions useful in general insurance work	68
5.1 The normal distribution	68
5.2 The Central Limit Theorem	77

Contents

5.3	The log-normal distribution	81
*5.4	The Pareto distribution	84
*5.5	The gamma distribution	86
5.6	The Poisson distribution	87
5.7	Normal approximation to the Poisson distribution	91
*5.8	The binomial distribution	93
**5.9	The negative binomial distribution; heterogeneity of risk	96
5.10	The importance of theoretical distributions in general insurance	101
5.11	Exercises	102
6	Inferences from general insurance data	103
6.1	Hypothesis testing	103
6.2	Point estimation and method of moments	107
*6.3	Maximum likelihood	108
*6.4	Confidence intervals	110
*6.5	Risk factors; multivariate models; least squares	112
6.6	Exercises	120
7	The risk premium	122
7.1	Risk premium; claim frequency and claim size	122
7.2	Claim frequency rate; exposure	123
7.3	Claim size; pitfalls	128
7.4	Claim settlement pattern	137
*7.5	Excesses and excess of loss reinsurances	145
7.6	Exercises	150
8	Experience rating	151
8.1	Introduction	151
8.2	Credibility theory	156
8.3	Full credibility	157
8.4	Partial credibility	160
*8.5	Bayes' Theorem	161
**8.6	A Bayesian approach to the updating of claim frequency rates	162
8.7	No claim discount (NCD)	166
8.8	Exercises	175
9	Simulation	177
9.1	Random numbers and simulation	177
9.2	How many simulations?	182
9.3	Computer generation of random numbers	184
9.4	Linear congruential generators	184
9.5	Random observations on the normal distribution	186
9.6	Random observations on the log-normal distribution	188
9.7	Random observations on the Poisson distribution	189
*9.8	Random observations on the negative binomial distribution	192
9.9	A simulation example	193
9.10	When to simulate	195
9.11	Simulation of an NCD system	198
9.12	Limitations of the model; sensitivity analysis	203
9.13	Exercises	204

Contents

10	Estimation of outstanding claim provisions	206
10.1	Delays in claim reporting and claim settlement; run-off	206
10.2	The run-off triangle	207
10.3	Chain-ladder method without inflation adjustment	208
10.4	Does the chain-ladder model fit the data?	209
10.5	Chain-ladder method with inflation adjustment	211
10.6	The separation method (direct future payments approach)	221
*10.7	The separation method (two other approaches)	229
10.8	IBNR, and the chain-ladder and separation methods	231
10.9	Alternative methods of assessing outstanding claim provisions	232
10.10	Estimation of IBNR claim provisions	232
10.11	Exercises	236
11	Elementary risk theory	238
11.1	Introduction	238
11.2	Portfolio with constant (fixed) claim size	240
11.3	Variable claim size	243
**11.4	The expectation and variance of the total claim amount C	250
11.5	The assumption of normality	250
11.6	Summary and further reading	252
11.7	Exercises	254
	References	255
	Solutions to exercises	257
	Author index	269
	Subject index	270