Contents

Editor's Statement . . . . . **xiii**
Foreword . . . . . **xv**
Preface . . . . . **xix**

Chapter 0 Elements of Functional Analysis **1**

1 Banach and Hilbert Spaces. Bounded Operators. Integration of Vector-Valued Functions 1
2 Linear Functionals: The Dual Space. Vector-Valued Analytic Functions 4
3 Unbounded Operators; the Resolvent. Closed Operators 6
4 Adjoints 11
5 Generalized Sequences. Weak Convergence 15
6 Normal, Symmetric, Self-Adjoint, and Unitary Operators in Hilbert Space 16
7 Some Special Spaces and Their Duals 20
8 Convolution and Mollifiers. Sobolev Spaces 23

Chapter 1 The Cauchy Problem for Some Equations of Mathematical Physics: The Abstract Cauchy Problem **26**

1.1 The Heat Equation in a Square 27
1.2 The Abstract Cauchy Problem 29

1.3 The Diffusion Equation in a Square 32
1.4 The Schrödinger Equation 39
1.5 The Cauchy Problem in $(-\infty, \infty)$ 43
1.6 The Maxwell Equations 43
1.7 Miscellaneous Notes 54

Chapter 2 Properly Posed Cauchy Problems: General Theory 62

2.1 The Cauchy Problem in $t \geqslant 0$ 63
2.2 The Cauchy Problem in $-\infty < t < \infty$. The Adjoint Equation 71
2.3 Semigroup Theory 80
2.4 The Inhomogeneous Equation 86
2.5 Miscellaneous Comments 91

Chapter 3 Dissipative Operators and Applications 117

3.1 Dissipative Operators 117
3.2 Ordinary Differential Operators in the Whole Line 125
3.3 Ordinary Differential Operators in a Closed Interval. Semi-Infinite Intervals 135
3.4 Ordinary Differential Operators in a Compact Interval 142
3.5 Symmetric Hyperbolic Systems in the Whole Space 146
3.6 Isometric Propagators and Conservative Operators. Dissipative Operators in Hilbert Space 154
3.7 Differential Equations in Banach Lattices. Positive Solutions. Dispersive Operators 159
3.8 Miscellaneous Comments 166

Chapter 4 Abstract Parabolic Equations: Applications to Second Order Parabolic Equations 172

4.1 Abstract Parabolic Equations 173
4.2 Abstract Parabolic Equations; Analytic Propagators 179
4.3 Applications to Ordinary Differential Operators 187
4.4 Second Order Partial Differential Operators. Dissipativity 199
4.5 Second Order Partial Differential Operators. Assignation of Boundary Conditions 204
4.6 Second Order Partial Differential Operators. Construction of m-Dissipative Extensions in L^2 214
4.7 Regularity Theorems 222
4.8 Construction of m-Dissipative Extensions in $L^p(\Omega)$ and $C(\overline{\Omega})$ 235
4.9 Analyticity of Solution Operators 255

4.10 Positivity and Compactness of Solution Operators 263
4.11 Miscellaneous Comments 265

Chapter 5 Perturbation and Approximation of Abstract Differential Equations 267

5.1 A Perturbation Result 268
5.2 The Neutron Transport Equation 272
5.3 Perturbation Results for Operators in $\mathcal{C}(1,0)$ and in $\mathcal{A}$ 286
5.4 Perturbation Results for the Schrödinger and Dirac Equations 292
5.5 Second Order Differential Operators 304
5.6 Symmetric Hyperbolic Systems in Sobolev Spaces 306
5.7 Approximation of Abstract Differential Equations 316
5.8 Approximation of Abstract Differential Equations by Finite Difference Equations 329
5.9 Miscellaneous Comments 336

Chapter 6 Some Improperly Posed Cauchy Problems 346

6.1 Improperly Posed Problems 346
6.2 The Reversed Cauchy Problem for Abstract Parabolic Equations 354
6.3 Fractional Powers of Certain Unbounded Operators 357
6.4 Fractional Powers of Certain Unbounded Operators (Continuation) 365
6.5 An Application: The Incomplete Cauchy Problem 372
6.6 Miscellaneous Comments 374

Chapter 7 The Abstract Cauchy Problem for Time-Dependent Equations 381

7.1 The Abstract Cauchy Problem for Time-Dependent Equations 381
7.2 Abstract Parabolic Equations 390
7.3 Abstract Parabolic Equations: Weak Solutions 405
7.4 Abstract Parabolic Equations: The Analytic Case 409
7.5 The Inhomogeneous Equation 415
7.6 Parabolic Equations with Time-Dependent Coefficients . . . 419
7.7 The General Case 424
7.8 Time-Dependent Symmetric Hyperbolic Systems in the Whole Space 437
7.9 The Case where $D(A(t))$ is Independent of t. The Inhomogeneous Equation 451
7.10 Miscellaneous Comments 455

Chapter 8 The Cauchy Problem in the Sense of Vector-Valued Distributions 461

8.1 Vector-Valued Distributions. Supports, Convergence, Structure Results 461
8.2 Vector-Valued Distributions. Convolution, Tempered Distributions, Laplace Transforms 468
8.3 Convolution and Translation Invariant Operators. Systems: The State Equation 477
8.4 The Cauchy Problem in the Sense of Distributions 482
8.5 The Abstract Parabolic Case 493
8.6 Applications: Extensions of the Notion of Properly Posed Cauchy Problem 497
8.7 Miscellaneous Comments 504

References 510

Index 627