

CONTENTS

<i>Preface</i>	<i>page</i>	ix
Further information		xiv
1 Axiomatic motivation of vector addition		1
Exercises and further results		8
2 Cauchy's equation. Hamel basis		11
2.1 General considerations, extensions, and regular solutions		11
2.2 General solutions		18
Exercises and further results		22
3 Three further Cauchy equations. An application to information theory		25
Exercises and further results		31
4 Generalizations of Cauchy's equations to several multiplace vector and matrix functions. An application to geometric objects		34
4.1 Multiplace and vector functions		34
4.2 A matrix functional equation and a characterization of densities in the theory of geometric objects		38
4.3 Pexider equations		42
4.4 Cauchy-type equations on semigroups		46
Exercises and further results		48
5 Cauchy's equations for complex functions. Applications to harmonic analysis and to information measures		52
5.1 Cauchy's equation and the exponential equation for complex functions		52
5.2 Endomorphisms of the real and complex fields		57
5.3 Bohr groups		60
5.4 Recursive entropies		66
Exercises and further results		69
6 Conditional Cauchy equations. An application to geometry and a characterization of the Heaviside functions		73
Exercises and further results		82

7 Addundancy, extensions, quasi-extensions and extensions almost everywhere. Applications to harmonic analysis and to rational decision making	84
7.1 Extensions and quasi-extensions	84
7.2 Extensions almost everywhere and integral transforms	92
7.3 Consensus allocations	99
Exercises and further results	101
8 D'Alembert's functional equation. An application to noneuclidean mechanics	103
Exercises and further results	111
9 Images of sets and functional equations. Applications to relativity theory and to additive functions bounded on particular sets	114
9.1 Equations containing images of sets and chronogeometry	114
9.2 Sets on which bounded additive functions are continuous	121
Exercises and further results	127
10 Some applications of functional equations in functional analysis, in the geometry of Banach spaces and in valuation theory	129
10.1 Functional equations and extreme points	129
10.2 Totally monotonic functions and extreme rays	135
10.3 A characterization of strictly convex normed spaces	138
10.4 Isometries in real normed spaces	142
10.5 A topology on the set of all solutions of a functional equation: the Bohr group	148
10.6 Valuations on the fields of rational and of real numbers	156
Exercises and further results	161
11 Characterizations of inner product spaces. An application to gas dynamics	165
11.1 Quadratic functionals: a characterization of inner product space	165
11.2 Triangles in normed spaces: a second characterization of inner product spaces	178
11.3 Orthogonal additivity	185
11.4 An application to gas dynamics	191
Exercises and further results	194
12 Some related equations and systems of equations. Applications to combinatorics and Markov processes	201
Exercises and further results	207
13 Equations for trigonometric and similar functions	209
Exercises and further results	225
14 A class of equations generalizing d'Alembert and Cauchy Pexider-type equations	228
Exercises and further results	237
15 A further generalization of Pexider's equation. A uniqueness theorem. An application to mean values	240
Exercises and further results	250

16 More about conditional Cauchy equations. Applications to additive number theoretical functions and to coding theory	254
16.1 Expansions of the Cauchy equation from curves	254
16.2 Cylindrical conditions	261
16.3 Additive number theoretical functions and related equations	265
16.4 An application to mean codeword lengths	267
16.5 Totally additive number theoretical functions and their generalizations	274
16.6 Further equations for number theoretical functions	281
Exercises and further results	283
17 Mean values, mediality and self-distributivity	287
Exercises and further results	296
18 Generalized mediality. Connection to webs and nomograms	298
Exercises and further results	307
19 Further composite equations. An application to averaging theory	309
19.1 One-parameter subgroups of affine groups	311
19.2 Another example of determining one-parameter subgroups	319
19.3 Two more composite equations	326
19.4 Reynolds and averaging operators	330
19.5 Interpolating and extension operators	334
19.6 Derivation operators	337
Exercises and further results	340
20 Homogeneity and some generalizations. Applications to economics	345
Exercises and further results	352
21 Historical notes	355
21.1 Definition of linear and quadratic functions by functional equations in the Middle Ages and application of an implied characterization by Galileo	355
21.2 The functional equations of the logarithm and of the exponential function	360
21.3 Some functional equations in the works of Euler	362
21.4 Functional equations arising from physics	363
21.5 The binomial theorem and Cauchy's equations	365
21.6 Cauchy equations after Cauchy	371
21.7 Further equations	373
21.8 Recent developments	377
Notations and symbols	379
Hints to selected 'exercises and further results'	382
Bibliography	388
Author index	449
Subject index	458