

Contents

Preface	ix
PART I VARIATIONAL PROBLEMS	
Chapter 1 Introduction	3
1.1 Example of a variational problem. The notion of well-posed problem	3
1.2 Some existence results	6
Chapter 2 Minimization of Convex Functionals	9
2.1 Fundamental theorem	9
2.2 Variational formulation of a minimization problem	15
2.3 Projections on convex sets	18
Chapter 3 Variational Inequalities	21
3.1 Fundamental theorem	21
3.2 Minimization of convex functionals	30
3.2.1 Gâteaux derivative	30
3.2.2 General results	34
3.2.3 The concept of subdifferential	37
Chapter 4 Transposition of Operators. Applications	41
4.1 The concept of transposition. Fundamental properties	41
4.2 Applications of the concept of transposition	45
4.2.1 Some examples of function spaces important in the applications. Examples of continuous linear operators	47
4.2.2 Dual spaces and transposed operators	55
Chapter 5 Sobolev Spaces	67
5.1 On the necessity for new function spaces	67
5.2 Spaces $W^{s,p}(\mathbb{R}^n)$	69
5.3 Spaces $W^{s,p}(\Omega)$	84
5.4 Spaces $W^{s,p}(\Gamma)$	93
5.5 Normal contractions and Dirichlet spaces	99

Chapter 6	Examples of Variational Problems in One Dimension . . .	104
6.1	The obstacle problem. Generalities about regularity results . . .	104
6.2	Some considerations regarding second order linear problems . . .	112
Chapter 7	Examples of Variational Problems in Several Dimensions	119
7.1	General comments on differential operators	119
7.2	Linear problems	129
7.2.1	Introduction. The homogeneous Dirichlet problem	129
7.2.2	General formulation in variational terms	137
7.2.3	Examples of boundary value problems	143
7.3	Nonlinear problems	156
7.4	Regularity results	169
7.4.1	Regularity of the solutions of differential equations	169
7.4.2	Regularity of solutions of variational inequalities	174
Chapter 8	Variational Formulation of a Free-Boundary Problem	183
8.1	Generalities. The physical problem	183
8.2	Transformation of the problem	187
8.3	Quasivariational problems	190

PART II QUASIVARIATIONAL PROBLEMS

Chapter 9	Fixed Point Theorems	199
9.1	Introduction	199
9.2	Fixed point theorems for Lipschitz continuous applications	201
9.2.1	Single-valued applications: Banach theorem	201
9.2.2	Multi-valued applications	204
9.3	Fixed point theorems for continuous applications	205
9.3.1	The Knaster–Kuratowski–Mazurkiewicz lemma and the Fan lemma	205
9.3.2	Single-valued applications: the theorems of Brouwer, Schauder and Tychonov	210
9.3.3	Multi-valued applications	216
9.4	Fixed point theorems for monotone applications	222
Chapter 10	Some Results on the Existence of Solutions of Variational Inequalities	226
10.1	General results of existence	226
10.2	Particular cases. I	232
10.3	Particular cases. II	234
Chapter 11	Quasivariational Inequalities	237
11.1	Introduction	237

11.2	Techniques of monotonicity	244
11.3	Techniques of compactness	253
Chapter 12 Free-Boundary Problems		262
12.1	Introduction	262
12.2	The physical problem	264
12.3	The mathematical problem	276
12.3.1	Rigorous formulation of the free-boundary problem	277
12.3.2	Transformation of the free-boundary problem	280
12.3.3	Study of a linear mixed problem	286
12.3.4	Study of a nonlinear mixed problem	291
12.3.5	Study of a quasivariational problem. Existence of the solution of a free-boundary problem	294
12.3.6	Uniqueness and regularity of the solution of a free- boundary problem	302
12.3.7	Dam with vertical walls	304
Chapter 13 Free-Boundary Problems and Variational Inequalities		317
PART III TECHNICAL TOOLS		
Chapter 14 Seminorms		325
Chapter 15 Regularization and Partition of the Unit		335
15.1	Regularization	335
15.2	Partition of the unit	342
Chapter 16 On the Regularity of the Open Sets		346
16.1	Manifolds. Open sets of class C^k and $C^{k,\mu}$. Cone and segment properties	346
16.2	Distributions on a manifold. Spaces $L^p(\Gamma)$	353
Chapter 17 The Maximum Principle and its Applications		357
17.1	Introduction	357
17.2	Maximum principle in \mathbb{R}	357
17.3	Maximum principle in \mathbb{C}	361
17.4	Maximum principle in \mathbb{R}^n	365
17.5	Complements	368
Chapter 18 On Green's Formulae		370
Chapter 19 Ordered Structures		381
19.1	Basic definitions	381
19.2	Lattices	384
19.3	Ordered vector spaces. Vector lattices	388

19.4	Topological vector lattices. Banach–Riesz spaces	393
19.5	Hilbert pseudo-lattices	399
Chapter 20	Multi-Valued Mappings	401
20.0	Notations	401
20.1	Fundamental definitions	401
20.2	Topologies in 2^Y	405
20.2.1	The Hausdorff metric topology	405
20.2.2	Victoris topologies	408
20.3	Orders in 2^Y_i	414
Bibliography	417
Index	449
Special Function Spaces Index	452