

# CONTENTS

Preface .....	V
Preface to the English edition .....	X
Contents .....	XI
Notation .....	XV
<b>CHAPTER 1. LIE ALGEBRAS .....</b>	<b>1</b>
1.1. General remarks .....	1
1.2. Representations .....	4
1.3. Solvable and nilpotent Lie algebras .....	11
1.4. The radical. The largest nilpotent ideal .....	17
1.5. Semi-simple Lie algebras .....	19
1.6. Semi-simplicity of representations .....	21
1.7. Reductive Lie algebras .....	26
1.8. Representations of $\mathfrak{sl}(2, k)$ .....	31
1.9. Cartan subalgebras .....	33
1.10. The system of roots of a split semi-simple Lie algebra .....	37
1.11. Regular linear forms .....	46
1.12. Polarizations .....	50
1.13. Symmetric semi-simple Lie algebras .....	57
1.14. Supplementary remarks .....	62
<b>CHAPTER 2. ENVELOPING ALGEBRAS .....</b>	<b>66</b>
2.1. The Poincaré–Birkhoff–Witt theorem .....	66
2.2. The functor $U$ .....	70
2.3. The filtration of the enveloping algebra .....	75
2.4. The canonical mapping of the symmetric algebra into the enveloping algebra .....	77
2.5. The existence of finite-dimensional representations .....	82
2.6. The commutant of a simple module .....	85
2.7. The dual of the enveloping algebra .....	89
2.8. Supplementary remarks .....	96

CHAPTER 3.	TWO-SIDED IDEALS IN ENVELOPING ALGEBRAS . . . . .	101
3.1.	Primitive ideals and prime ideals . . . . .	101
3.2.	The space of primitive ideals . . . . .	105
3.3.	The passage to an ideal of $\mathfrak{g}$ . . . . .	107
3.4.	Extension of the scalar field . . . . .	111
3.5.	The Krull dimension . . . . .	112
3.6.	Rings of fractions . . . . .	117
3.7.	Prime ideals in the solvable case . . . . .	125
3.8.	Supplementary remarks . . . . .	128
CHAPTER 4.	CENTRES . . . . .	131
4.1.	Notation . . . . .	131
4.2.	Centre and core in the semi-simple case . . . . .	133
4.3.	The semi-centre . . . . .	134
4.4.	Centre and core in the solvable case . . . . .	135
4.5.	The characterization of primitive ideals in the solvable case . . . . .	141
4.6.	Heisenberg and Weyl algebras . . . . .	146
4.7.	Centre and core in the nilpotent case . . . . .	151
4.8.	Invariant ideals of the symmetric algebra (the nilpotent case) . . . . .	158
4.9.	Supplementary remarks . . . . .	162
CHAPTER 5.	INDUCED REPRESENTATIONS . . . . .	169
5.1.	Induced representations . . . . .	169
5.2.	Twisted induced representations . . . . .	174
5.3.	A criterion for the simplicity of induced representations . . . . .	176
5.4.	The construction of primitive ideals by induction . . . . .	180
5.5.	Co-induced representations . . . . .	186
5.6.	Supplementary remarks . . . . .	188
CHAPTER 6.	PRIMITIVE IDEALS (THE SOLVABLE CASE) . . . . .	192
6.1.	The ideals $I(f)$ . . . . .	192
6.2.	Rational ideals in the nilpotent case . . . . .	199
6.3.	Prime ideals of the enveloping algebra and invariant prime ideals of the symmetric algebra (the nilpotent case) . . . . .	204
6.4.	The Jacobson topology . . . . .	207
6.5.	The injectivity of the mapping $\bar{I}$ . . . . .	215
6.6.	Supplementary remarks . . . . .	227
CHAPTER 7.	VIRMA MODULES . . . . .	231
7.0.	Notation . . . . .	231
7.1.	The modules $L(\lambda)$ and $M(\lambda)$ . . . . .	232
7.2.	Finite-dimensional representations . . . . .	236
7.3.	Invariants in the symmetric algebra . . . . .	239
7.4.	The Harish-Chandra homomorphism . . . . .	242
7.5.	Characters . . . . .	246

7.6.	Submodules of $M(\lambda)$ .....	249
7.7.	Submodules of $M(\lambda)$ and the ordering relation on the Weyl group .....	264
7.8.	Supplementary remarks .....	267
<b>CHAPTER 8.</b>	<b>THE ENVELOPING ALGEBRA OF A SEMI-SIMPLE LIE ALGEBRA .....</b>	<b>277</b>
8.1.	The cone of nilpotent elements .....	277
8.2.	The enveloping algebra as a module over its centre .....	281
8.3.	The adjoint representation in the enveloping algebra .....	283
8.4.	Annihilators of Verma modules .....	288
8.5.	Supplementary remarks .....	291
<b>CHAPTER 9.</b>	<b>HARISH-CHANDRA MODULES .....</b>	<b>295</b>
9.0.	Notation .....	295
9.1.	The case of a Lie subalgebra which is reductive in $\mathfrak{g}$ .....	295
9.2.	Canonical mappings defined by a symmetrizing subalgebra .....	301
9.3.	The principal series .....	308
9.4.	The subquotient theorem .....	310
9.5.	Finiteness theorems .....	313
9.6.	Spherical modules in the diagonal case .....	315
9.7.	Supplementary remarks .....	321
<b>CHAPTER 10.</b>	<b>PRIMITIVE IDEALS (THE GENERAL CASE) .....</b>	<b>325</b>
10.1.	Some canonical homomorphisms .....	325
10.2.	Application to induced representations .....	331
10.3.	The ideals $I(f)$ .....	334
10.4.	Application to the centre of the enveloping algebra .....	341
10.5.	Supplementary remarks .....	344
<b>CHAPTER 11.</b>	<b>APPENDIX .....</b>	<b>346</b>
11.1.	Root systems .....	346
11.2.	Miscellaneous results .....	350
<b>PROBLEMS .....</b>		<b>354</b>
<b>BIBLIOGRAPHY .....</b>		<b>359</b>
<b>INDEX .....</b>		<b>371</b>