

CONTENTS

Chapter 1	Elementary Definitions and Examples	1
1.	Semigroups and Monoids, 1	
2.	Examples: Transformation Semigroups, Free Semigroups, 4	
3.	Equivalences on Sets. Congruences on Semigroups, 9	
4.	Presentations of Semigroups and Monoids, 11	
Chapter 2	Green's Relations	19
1.	Elementary Ideal Theory, 19	
2.	Definitions and General Properties of Green's Relations, 25	
3.	The Schützenberger Group of a \mathcal{D} -Class, 31	
4.	Examples of Computations of Green's Relations, 38	
5.	Representations of a Semigroup Related to Green's Relations, 43	
Chapter 3	Simple Semigroups Rees–Suschkewitsch Theorem	53
1.	Completely Simple and Completely 0-Simple Semigroups, 54	
2.	\mathcal{D} -Representation of a Completely 0-Simple Semigroup. The Rees–Suschkewitsch Theorem, 56	
3.	Some Examples, 62	
4.	Determination of Regular \mathcal{D} -Classes in a Transformation Semigroup on a Finite Set, 64	
5.	Small Monoids and Their Relationships to Combinatorics, 68	

Chapter 4	The Prime Decomposition Theorem for Finite Monoids	78
1.	Preliminaries in Group Theory, 79	
2.	Decompositions of Finite Monoids, 83	
3.	The Prime Decomposition Theorem, 87	
4.	Irreducible Monoids, 90	
5.	Decompositions of Finite Transformation Monoids, 91	
Chapter 5	Free Monoids, Languages, and Codes	100
1.	Generalities on Free Monoids. Examples, 101	
2.	Submonoids of Free Monoids. Codes, 106	
3.	Elementary Algebraic Properties of Codes, 116	
4.	Preliminaries to the Word Problem. Recursion, Algorithms, Productions of Languages, 120	
5.	The Word Problem for Monoids and Related Questions, 130	
6.	Further Results and References, 135	
Chapter 6	Automata, Rational Languages, and Their Syntactic Monoids	142
1.	Automata, Recognizable Languages. Definitions and Elementary Properties, 143	
2.	Nondeterministic Automata. Languages of Right Linear Grammars, 152	
3.	Rational Subsets of a Monoid. Kleene's Theorem, 156	
4.	Rational Codes. Recurrent Events, 160	
5.	Pseudovarieties of Monoids and Streams of Languages, 165	
Chapter 7	Star Problems for Rational Languages	180
1.	Aperiodic Languages, 181	
2.	Languages Defined by Abelian Groups, 192	
3.	Languages Defined by Solvable Groups, 203	
Chapter 8	Rational Prefix Codes	211
1.	Finite Prefix Codes: Group of Units in Their Syntactic Monoids, 212	

2. Suschkewitsch Groups of Rational Prefix Codes, 213
3. Automata Congruences and Decompositions of Prefix Codes, 218
4. Suschkewitsch Groups of Complete Prefix Codes, 222
5. Decompositions of Finite Complete Prefix Codes, 229
6. Finite Complete Biprefix Codes, 234

Chapter 9 Algebraic Languages **249**

1. Context-Free Grammars, Examples, Reductions, 249
2. Closure Properties of Algebraic Languages. Rational Transductions, 255
3. Decidability Problems. Ambiguity, 259
4. Rational and Algebraic Languages over Rings and Semirings, 262
5. Abstract Families of Languages, 275

Chapter 10 The Burnside Problems and Related Questions **289**

1. A Factorization Lemma for Free Semigroups, 290
2. Decidability of the Linear Burnside Problem, 293
3. Semigroups $B(k, 1, n)$, 300
4. Semigroups $B(k, m, n)$ for $k > 1, m > 1$, 303
5. Infinite Sequences Without Repeats, 307

Chapter 11 Rearrangements, Factorizations, and MacMahon's Master Theorem **314**

1. The Master Theorem. Applications, 315
2. Flows and Rearrangements. Free Partially Commutative Monoids, 320
3. Möbius Function and Proof of the Master Theorem, 328
4. A Brief Survey of Elementary Lie Algebra Theory, 332
5. Factorizations of Free Monoids, 337
6. Examples of Factorizations, 347

References **357**

Symbols **369**

Index **373**