CONTENTS

INTRODUCTION v

REMERCIEMENTS vii

PART I

SUMMARY OF CHAPTER I 3

CHAPTER I. RADON MEASURES: MAIN PROPERTIES AND OPERATIONS

1. Preliminaries 7
2. Definition of Radon measure on general topological spaces 12
3. Induced measures 20
4. Product of a measure by a locally integrable function 23
5. Measurable mappings and image measures . . 25
6. Other properties of Radon measures 40
7. Real and complex measures 53
8. Different ways of defining Radon measures . . 58
9. Tensor product of measures 63
10. Projective limit of measures 74
11. Non-Hausdorff spaces 82

SUMMARY OF CHAPTER II 89

CHAPTER II. POLISH, LUSIN, SUSLIN AND RADON SPACES

1. Definitions and topological properties . . . 92
2. Examples of Lusin spaces 112
3. Radon spaces 117
4. Lifting of L^∞ to $\mathscr{L}^\infty$ 130
5. Spaces of regulated functions 136

TERMINOLOGICAL INDEX TO PART I 144

REFERENCES 146

PART II

INTRODUCTION TO PART II 151

SUMMARY OF CHAPTER I 152

CHAPTER I. EQUIVALENT TOPOLOGIES

1. Equivalent measures 153
2. Equivalent topologies 155
3. A Borel-graph theorem 160
4. Equivalence of the strong and weak topologies . 161
5. Equivalence of the strong topology with coarser topologies 166

SUMMARY OF CHAPTER II 171

CHAPTER II. CYLINDRICAL MEASURES

1. Definition of cylindrical measures . . . 172
2. Operations with cylindrical measures . . . 180
3. Concentrations 188

SUMMARY OF CHAPTER III 206

CHAPTER III. CYLINDRICAL MEASURES AND HILBERT-SCHMIDT OPERATORS

1. Hilbert-Schmidt operators 207
2. The main lemma 212
3. The theorem of Sazonov 214

SUMMARY OF CHAPTER IV 218

CHAPTER IV. CYLINDRICAL MEASURES AND NUCLEAR SPACES

1. Nuclear operators 219
2. Nuclear spaces 226
3. The theorem of Minlos 233
4. The theorem of Sazonov-Badrikian 237

SUMMARY OF CHAPTER V 242

CHAPTER V. CYLINDRICAL MEASURES AND PROBABILITY THEORY

1. Basic concepts of probability theory 244
2. Randon functions 254
3. Examples and applications 269
4. An analogue of Prokhorov's theorem for random variables with values in a Suslin space . . . 288
5. Categories of random functions 305

SUMMARY OF CHAPTER VI 324

CHAPTER VI. GAUSS MEASURES

1. The definition of Gauss measures 325
2. The Hilbert subspaces of a locally convex space . 330
3. The converse of the theorems of Sazonov and Minlos 341
4. The analogue of Sazonov's theorem for l^p spaces $(0 < p \leqslant +\infty)$ and applications . . . 347
5. An application to Brownian motion 350

SUMMARY OF APPENDIX 368

APPENDIX. COMPLEMENTS ON NARROW CONVERGENCE

1. Narrow convergence on an arbitrary topological space 369
2. Riemann-integrable functions 374

3. A compactness criterion for the narrow topology . 379
4. A theorem of Paul Lévy 382
5. The space of finite measures on a Suslin space . 385

TERMINOLOGICAL INDEX TO PART II 390

REFERENCES 392