

# Contents

---

<b>Introduction</b>	<i>page</i> 1
<b>1 Classical Lie algebras and Weyl groups</b>	5
1.1 Lie algebras	5
1.2 The classical Lie algebras	8
1.3 Classical Lie algebras and partially ordered sets	11
1.4 Classical Weyl groups and partially ordered sets	16
1.5 Notes and references	18
<b>2 Heaps over graphs</b>	19
2.1 Basic definitions	20
2.2 Full heaps over Dynkin diagrams	26
2.3 Local structure of full heaps	30
2.4 Quotient heaps	38
2.5 Notes and references	40
<b>3 Weyl group actions</b>	42
3.1 Linear operators and group actions	43
3.2 Proper ideals	55
3.3 Parabolic subheaps	62
3.4 Notes and references	67
<b>4 Lie theory</b>	70
4.1 Representations of Lie algebras from heaps	70
4.2 Review of Lie theory	75
4.3 Review of Weyl groups	80
4.4 Strongly orthogonal sets	86
4.5 Notes and references	88
<b>5 Minuscule representations</b>	89
5.1 Highest weight modules	89
5.2 Weights and heaps	92
5.3 Periodicity and trivialization	96
5.4 Reflections	107
5.5 Minuscule representations from heaps	112

5.6	Invariant bilinear forms	115
5.7	Notes and references	116
<b>6</b>	<b>Full heaps over affine Dynkin diagrams</b>	<b>118</b>
6.1	Full heaps in type $A_l^{(1)}$	118
6.2	Proper ideals in type $A_l^{(1)}$	125
6.3	Spin representations in type $D_l$	129
6.4	Types $B_l^{(1)}$ and $D_{l+1}^{(2)}$	134
6.5	Full heaps in type $E_6^{(1)}$ and $E_7^{(1)}$	139
6.6	The classification of full heaps over affine Dynkin diagrams	144
6.7	Notes and references	147
<b>7</b>	<b>Chevalley bases</b>	<b>148</b>
7.1	Kac's asymmetry function	148
7.2	Relations in simply laced simple Lie algebras	153
7.3	Folding	160
7.4	Long and short roots	165
7.5	Relations in non-simply laced simple Lie algebras	175
7.6	Notes and references	181
<b>8</b>	<b>Combinatorics of Weyl groups</b>	<b>183</b>
8.1	Minuscule systems	183
8.2	Weyl groups as permutation groups	188
8.3	Ideals of roots	197
8.4	Weight polytopes	202
8.5	Faces of weight polytopes	207
8.6	Graphs from minuscule representations	211
8.7	Notes and references	214
<b>9</b>	<b>The 28 bitangents</b>	<b>216</b>
9.1	The Gosset graph	216
9.2	Del Pezzo surfaces	219
9.3	Bitangents	226
9.4	Hesse–Cayley notation	231
9.5	Steiner complexes	237
9.6	Symplectic structure	244
9.7	Notes and references	247
<b>10</b>	<b>Exceptional structures</b>	<b>248</b>
10.1	The 27 lines on a cubic surface	249
10.2	Combinatorics of double sixes	253
10.3	2-graphs	258
10.4	Generalized quadrangles	265
10.5	Higher invariant forms	269
10.6	Notes and references	273
<b>11</b>	<b>Further topics</b>	<b>275</b>
11.1	Minuscule elements of Weyl groups	275

11.2	Principal subheaps as abstract posets	280
11.3	Gaussian posets	284
11.4	Jeu de taquin	289
11.5	Notes and references	296
<b>Appendix A Posets, graphs and categories</b>		298
<b>Appendix B Lie theoretic data</b>		304
	<i>References</i>	307
	<i>Index</i>	311