

# Contents

<i>Preface</i> .....	xiii
<i>Conventions and Notation</i> .....	xvii
<b>1. Introduction to Wavelets</b> .....	<b>1</b>
1.0 Introduction .....	1
1.1 The Essence of a Wavelet .....	2
Comments and Extensions to Section 1.1 .....	4
1.2 The Essence of Wavelet Analysis .....	5
Comments and Extensions to Section 1.2 .....	12
1.3 Beyond the CWT: the Discrete Wavelet Transform .....	12
Comments and Extensions to Section 1.3 .....	19
<b>2. Review of Fourier Theory and Filters</b> .....	<b>20</b>
2.0 Introduction .....	20
2.1 Complex Variables and Complex Exponentials .....	20
2.2 Fourier Transform of Infinite Sequences .....	21
2.3 Convolution/Filtering of Infinite Sequences .....	24
2.4 Fourier Transform of Finite Sequences .....	28
2.5 Circular Convolution/Filtering of Finite Sequences .....	29
2.6 Periodized Filters .....	32
Comments and Extensions to Section 2.6 .....	35
2.7 Summary of Fourier Theory .....	35
2.8 Exercises .....	39

<b>3. Orthonormal Transforms of Time Series .....</b>	<b>41</b>
3.0 Introduction .....	41
3.1 Basic Theory for Orthonormal Transforms .....	41
3.2 The Projection Theorem .....	44
3.3 Complex-Valued Transforms .....	45
3.4 The Orthonormal Discrete Fourier Transform .....	46
Comments and Extensions to Section 3.4 .....	52
3.5 Summary .....	53
3.6 Exercises .....	54
<b>4. The Discrete Wavelet Transform .....</b>	<b>56</b>
4.0 Introduction .....	56
4.1 Qualitative Description of the DWT .....	57
Key Facts and Definitions in Section 4.1 .....	67
Comments and Extensions to Section 4.1 .....	68
4.2 The Wavelet Filter .....	68
Key Facts and Definitions in Section 4.2 .....	74
Comments and Extensions to Section 4.2 .....	75
4.3 The Scaling Filter .....	75
Key Facts and Definitions in Section 4.3 .....	78
Comments and Extensions to Section 4.3 .....	79
4.4 First Stage of the Pyramid Algorithm .....	80
Key Facts and Definitions in Section 4.4 .....	86
Comments and Extensions to Section 4.4 .....	87
4.5 Second Stage of the Pyramid Algorithm .....	88
Key Facts and Definitions in Section 4.5 .....	93
4.6 General Stage of the Pyramid Algorithm .....	93
Key Facts and Definitions in Section 4.6 .....	99
Comments and Extensions to Section 4.6 .....	100
4.7 The Partial Discrete Wavelet Transform .....	104
4.8 Daubechies Wavelet and Scaling Filters: Form and Phase .....	105
Key Facts and Definitions in Section 4.8 .....	116
Comments and Extensions to Section 4.8 .....	117
4.9 Coiflet Wavelet and Scaling Filters: Form and Phase .....	123
4.10 Example: Electrocardiogram Data .....	125
Comments and Extensions to Section 4.10 .....	134
4.11 Practical Considerations .....	135
Comments and Extensions to Section 4.11 .....	145
4.12 Summary .....	150
4.13 Exercises .....	156

<b>5. The Maximal Overlap Discrete Wavelet Transform .....</b>	<b>159</b>
5.0 Introduction .....	159
5.1 Effect of Circular Shifts on the DWT .....	160
5.2 MODWT Wavelet and Scaling Filters .....	163
5.3 Basic Concepts for MODWT .....	164
Key Facts and Definitions in Section 5.3 .....	168
5.4 Definition of $j$ th Level MODWT Coefficients .....	169
Key Facts and Definitions in Section 5.4 .....	173
Comments and Extensions to Section 5.4 .....	174
5.5 Pyramid Algorithm for the MODWT .....	174
Key Facts and Definitions in Section 5.5 .....	177
Comments and Extensions to Section 5.5 .....	177
5.6 MODWT Analysis of 'Bump' Time Series .....	179
5.7 Example: Electrocardiogram Data .....	182
5.8 Example: Subtidal Sea Level Fluctuations .....	185
5.9 Example: Nile River Minima .....	190
5.10 Example: Ocean Shear Measurements .....	193
5.11 Practical Considerations .....	195
5.12 Summary .....	200
5.13 Exercises .....	204
<b>6. The Discrete Wavelet Packet Transform .....</b>	<b>206</b>
6.0 Introduction .....	206
6.1 Basic Concepts .....	207
Comments and Extensions to Section 6.1 .....	217
6.2 Example: DWPT of Solar Physics Data .....	218
6.3 The Best Basis Algorithm .....	221
Comments and Extensions to Section 6.3 .....	226
6.4 Example: Best Basis for Solar Physics Data .....	226
6.5 Time Shifts for Wavelet Packet Filters .....	229
Comments and Extensions to Section 6.5 .....	231
6.6 Maximal Overlap Discrete Wavelet Packet Transform .....	231
6.7 Example: MODWPT of Solar Physics Data .....	234
6.8 Matching Pursuit .....	239
6.9 Example: Subtidal Sea Levels .....	243
Comments and Extensions to Section 6.9 .....	247
6.10 Summary .....	247
6.11 Exercises .....	253
<b>7. Random Variables and Stochastic Processes .....</b>	<b>255</b>
7.0 Introduction .....	255
7.1 Univariate Random Variables and PDFs .....	256
7.2 Random Vectors and PDFs .....	258
7.3 A Bayesian Perspective .....	264
7.4 Stationary Stochastic Processes .....	266
7.5 Spectral Density Estimation .....	269

Comments and Extensions to Section 7.5 .....	278
7.6 Definition and Models for Long Memory Processes .....	279
Comments and Extensions to Section 7.6 .....	285
7.7 Nonstationary $1/f$ -Type Processes .....	287
Comments and Extensions to Section 7.7 .....	289
7.8 Simulation of Stationary Processes .....	290
Comments and Extensions to Section 7.8 .....	292
7.9 Simulation of Stationary Autoregressive Processes .....	292
7.10 Exercises .....	293
<b>8. The Wavelet Variance .....</b>	<b>295</b>
8.0 Introduction .....	295
8.1 Definition and Rationale for the Wavelet Variance .....	295
Comments and Extensions to Section 8.1 .....	301
8.2 Basic Properties of the Wavelet Variance .....	304
Comments and Extensions to Section 8.2 .....	306
8.3 Estimation of the Wavelet Variance .....	306
Comments and Extensions to Section 8.3 .....	308
8.4 Confidence Intervals for the Wavelet Variance .....	311
Comments and Extensions to Section 8.4 .....	315
8.5 Spectral Estimation via the Wavelet Variance .....	315
Comments and Extensions to Section 8.5 .....	317
8.6 Example: Atomic Clock Deviates .....	317
8.7 Example: Subtidal Sea Level Fluctuations .....	324
8.8 Example: Nile River Minima .....	326
8.9 Example: Ocean Shear Measurements .....	327
8.10 Summary .....	335
8.11 Exercises .....	337
<b>9. Analysis and Synthesis of Long Memory Processes .....</b>	<b>340</b>
9.0 Introduction .....	340
9.1 Discrete Wavelet Transform of a Long Memory Process .....	341
Comments and Extensions to Section 9.1 .....	350
9.2 Simulation of a Long Memory Process .....	355
Comments and Extensions to Section 9.2 .....	361
9.3 MLEs for Stationary FD Processes .....	361
Comments and Extensions to Section 9.3 .....	366
9.4 MLEs for Stationary or Nonstationary FD Processes .....	368
Comments and Extensions to Section 9.4 .....	373
9.5 Least Squares Estimation for FD Processes .....	374
Comments and Extensions to Section 9.5 .....	378
9.6 Testing for Homogeneity of Variance .....	379
Comments and Extensions to Section 9.6 .....	382
9.7 Example: Atomic Clock Deviates .....	383
9.8 Example: Nile River Minima .....	386
9.9 Summary .....	388

9.10 Exercises .....	391
<b>10. Wavelet-Based Signal Estimation .....</b>	<b>393</b>
10.0 Introduction .....	393
10.1 Signal Representation via Wavelets .....	394
10.2 Signal Estimation via Thresholding .....	398
10.3 Stochastic Signal Estimation via Scaling .....	407
10.4 Stochastic Signal Estimation via Shrinkage .....	408
Comments and Extensions to Section 10.4 .....	415
10.5 IID Gaussian Wavelet Coefficients .....	417
Comments and Extensions to Section 10.5 .....	429
10.6 Uncorrelated Non-Gaussian Wavelet Coefficients .....	432
Comments and Extensions to Section 10.6 .....	439
10.7 Correlated Gaussian Wavelet Coefficients .....	440
Comments and Extensions to Section 10.7 .....	449
10.8 Clustering and Persistence of Wavelet Coefficients .....	450
10.9 Summary .....	452
10.10 Exercises .....	455
<b>11. Wavelet Analysis of Finite Energy Signals .....</b>	<b>457</b>
11.0 Introduction .....	457
11.1 Translation and Dilation .....	457
11.2 Scaling Functions and Approximation Spaces .....	459
Comments and Extensions to Section 11.2 .....	462
11.3 Approximation of Finite Energy Signals .....	462
Comments and Extensions to Section 11.3 .....	464
11.4 Two-Scale Relationships for Scaling Functions .....	464
11.5 Scaling Functions and Scaling Filters .....	469
Comments and Extensions to Section 11.5 .....	472
11.6 Wavelet Functions and Detail Spaces .....	472
11.7 Wavelet Functions and Wavelet Filters .....	476
11.8 Multiresolution Analysis of Finite Energy Signals .....	478
11.9 Vanishing Moments .....	483
Comments and Extensions to Section 11.9 .....	486
11.10 Spectral Factorization and Filter Coefficients .....	487
Comments and Extensions to Section 11.10 .....	494
11.11 Summary .....	494
11.12 Exercises .....	500
<b>Appendix. Answers to Embedded Exercises .....</b>	<b>501</b>
<b>References .....</b>	<b>552</b>
<b>Author Index .....</b>	<b>565</b>
<b>Subject Index .....</b>	<b>569</b>