CONTENTS

PREFACE ix

NOTE TO THE READER xi

1. INTRODUCTION 1

- 2. NONLINEAR DIFFERENTIAL EQUATIONS 6
- 2.1 Mathematical Preliminaries 6
- 2.2 Induced Norms and Matrix Measures 19
- 2.3 Contraction Mapping Theorem 27
- 2.4 Nonlinear Differential Equations 33
- 2.5 Solution Estimates 46

3. SECOND-ORDER SYSTEMS 53

- 3.1 Preliminaries 53
- 3.2 Linearization Method 57
- 3.3 Periodic Solutions 67
- 3.4 Two Analytical Approximation Methods 79

4. APPROXIMATE ANALYSIS METHODS 88

- 4.1 Describing Functions 88
- 4.2 Periodic Solutions: Rigorous Arguments 109
- 4.3 Singular Perturbations 127

5. LYAPUNOV STABILITY 135

- 5.1 Stability Definitions 135
- 5.2 Some Preliminaries 147
- 5.3 Lyapunov's Direct Method 157
- 5.4 Stability of Linear Systems 193
- 5.5 Lyapunov's Linearization Method 209
- 5.6 The Lur'e Problem 219
- 5.7 Converse Theorems 235

- 5.8 Applications of Converse Theorems 246
- 5.9 Discrete-Time Systems 264

6. INPUT-OUTPUT STABILITY 270

- 6.1 L_p -Spaces and their Extensions 271
- 6.2 Definitions of Input-Output Stability 277
- 6.3 Relationships Between I/O and Lyapunov Stability 284
- 6.4 Open-Loop Stability of Linear Systems 292
- 6.5 Linear Time-Invariant Feedback Systems 309
- 6.6 Time-Varying and/or Nonlinear Systems 337
- 6.7 Discrete-Time Systems 365

7. DIFFERENTIAL GEOMETRIC METHODS 376

- 7.1 Basics of Differential Geometry 377
- 7.2 Distributions, Frobenius Theorem 392
- 7.3 Reachability and Observability 399
- 7.4 Feedback Linearization: Single-Input Case 427
- 7.5 Feedback Linearization: Multi-Input Case 438
- 7.6 Input-Output Linearization 456
- 7.7 Stabilization of Linearizable Systems 464

A. PREVALENCE OF DIFFERENTIAL EQUATIONS WITH UNIQUE SOLU-TIONS 469

B. PROOF OF THE KALMAN-YACUBOVITCH LEMMA 474

C. PROOF OF THE FROBENIUS THEOREM 476

REFERENCES 486

INDEX 493