

Contents

| | |
|--|------|
| Preface | ix |
| Acknowledgments | xi |
| Introduction | xiii |
| | |
| 1. COLOMBEAU'S THEORY OF GENERALIZED FUNCTIONS | 1 |
| 1.1 Multiplication of Distributions | 1 |
| 1.2 The Special Algebra | 8 |
| 1.2.1 Definition and Basic Properties | 8 |
| 1.2.2 Embedding of $\mathcal{D}'(\Omega)$ | 16 |
| 1.2.3 Tempered Generalized Functions | 25 |
| 1.2.4 Point Values and Generalized Numbers | 31 |
| 1.2.5 Integration | 43 |
| 1.2.6 Association and Coupled Calculus | 47 |
| 1.3 A General Scheme of Construction | 54 |
| 1.4 The Full Colombeau Algebra | 57 |
| 1.4.1 Construction of the Algebra | 58 |
| 1.4.2 Point Values, Integration, Association | 66 |
| 1.4.3 Additional Constructions | 70 |
| 1.5 Applications to Differential Equations | 80 |
| 1.5.1 Existence and Uniqueness of Solutions | 80 |
| 1.5.2 Delta Function Potentials in Classical Mechanics | 88 |
| 1.6 Colombeau's Original Approach | 94 |
| | |
| 2. DIFFEOMORPHISM INVARIANT COLOMBEAU THEORY | 101 |
| 2.1 Introduction | 101 |
| 2.2 Calculus | 107 |
| 2.2.1 Calculus on Convenient Vector Spaces | 107 |
| 2.2.2 A Completeness Theorem | 113 |
| 2.3 Fundamentals | 116 |
| 2.3.1 Notation and Terminology | 116 |

| | | |
|--------|--|-----|
| 2.3.2 | C- and J-Formalism | 117 |
| 2.3.3 | Calculus on $U_\varepsilon(\Omega)$ | 125 |
| 2.4 | Definitions and Basic Theorems | 132 |
| 2.5 | Characterization Results I | 138 |
| 2.5.1 | The Chain Rule Lemma | 139 |
| 2.5.2 | Characterization Theorems I | 143 |
| 2.6 | Stability under Differentiation | 151 |
| 2.7 | Characterization Results II | 152 |
| 2.7.1 | Extending Bounded Paths | 154 |
| 2.7.2 | Characterization Theorems II | 158 |
| 2.8 | Diffeomorphism Invariance and $\mathcal{G}^d(\Omega)$ | 168 |
| 2.9 | Sheaf Properties | 177 |
| 2.10 | Separating the Basic Definition from Testing | 179 |
| 2.11 | Differential Equations | 181 |
| 2.12 | Non-Injectivity of the Canonical Homomorphism from $\mathcal{G}^d(\Omega)$ into $\mathcal{G}^e(\Omega)$ | 183 |
| 2.13 | Classification of Smooth Colombeau Algebras between $\mathcal{G}^d(\Omega)$ and $\mathcal{G}^e(\Omega)$ | 196 |
| 2.13.1 | The Development from $\mathcal{G}^e(\Omega)$ to $\mathcal{G}^d(\Omega)$ | 196 |
| 2.13.2 | Classification of Test Objects | 198 |
| 2.13.3 | Classification of Full Smooth Colombeau Algebras | 200 |
| 2.14 | The Algebra \mathcal{G}^2 ; Classification Results | 206 |
| 2.15 | Concluding Remarks | 217 |
| 3. | GENERALIZED FUNCTIONS ON MANIFOLDS | 219 |
| 3.1 | Distributions on Manifolds | 220 |
| 3.1.1 | Introduction | 220 |
| 3.1.2 | Densities, Integration, Orientation | 222 |
| 3.1.3 | Test Fields and Distributions | 229 |
| 3.1.4 | Local Description and Global Structure | 233 |
| 3.1.5 | Orientable Manifolds, Distributional Geometry | 243 |
| 3.2 | The Special Algebra on Manifolds | 277 |
| 3.2.1 | Basic Properties, Point Value Characterization | 277 |
| 3.2.2 | Embeddings and Association | 283 |
| 3.2.3 | Generalized Sections of Vector Bundles | 289 |
| 3.2.4 | Generalized Functions Valued in a Manifold | 303 |
| 3.2.5 | Generalized Pseudo-Riemannian Geometry | 324 |
| 3.3 | The Full Algebra on Manifolds | 332 |
| 3.3.1 | Introduction | 332 |
| 3.3.2 | Smoothing Kernels and Basic Function Spaces | 335 |
| 3.3.3 | Construction of the Algebra, Localization | 343 |
| 3.3.4 | Embedding of Distributions and Smooth Functions | 348 |

| | |
|---|-----|
| 4. APPLICATIONS TO LIE GROUP ANALYSIS OF DIFFERENTIAL EQUATIONS | 353 |
| 4.1 Introduction | 353 |
| 4.1.1 Lie Transformation Groups | 354 |
| 4.1.2 Symmetries of Differential Equations | 358 |
| 4.1.3 Calculation of Symmetry Groups | 364 |
| 4.2 Transfer of Classical Symmetry Groups | 369 |
| 4.2.1 Factorization Properties | 370 |
| 4.2.2 Continuity Properties | 384 |
| 4.2.3 Associated and Distributional Symmetries | 385 |
| 4.3 Generalized Group Actions | 390 |
| 4.3.1 Generalized Transformation Groups | 390 |
| 4.3.2 Generalized Symmetries of Differential Equations | 393 |
| 4.4 Infinitesimal Criteria | 399 |
| 4.5 Group Invariant Generalized Functions | 408 |
| 5. APPLICATIONS TO GENERAL RELATIVITY | 415 |
| 5.1 Introduction | 415 |
| 5.2 Linear and Nonlinear Distributional Geometry in General Relativity | 419 |
| 5.3 Distributional Description of Impulsive Gravitational Waves | 432 |
| 5.3.1 Impulsive pp-Waves | 432 |
| 5.3.2 The Geodesic Equation for Impulsive pp-Waves | 439 |
| 5.3.3 Geodesic Deviation for Impulsive pp-Waves | 450 |
| 5.3.4 Distributional vs. Continuous Form of the Metric | 462 |
| Appendices | 473 |
| The Chain Rule for Higher Differentials | 473 |
| References | 479 |
| Author Index | 496 |
| Index of Notation | 499 |
| Index | 502 |