Contents

1 Aims of Simulation 1

1.1 The Tools, 2
1.2 Models, 2
1.3 Simulation as Experimentation, 4
1.4 Simulation in Inference, 4
1.5 Examples, 5
1.6 Literature, 12
1.7 Convention, 12
Exercises, 13

2 Pseudo-Random Numbers 14

2.1 History and Philosophy, 14
2.2 Congruential Generators, 20
2.3 Shift-Register Generators, 26
2.4 Lattice Structure, 33
2.5 Shuffling and Testing, 42
2.6 Conclusions, 45
2.7 Proofs, 46
Exercises, 50

3 Random Variables 53

3.1 Simple Examples, 54
3.2 General Principles, 59
3.3 Discrete Distributions, 71
3.4 Continuous Distributions, 81
3.5 Recommendations, 91
Exercises, 92

4 Stochastic Models **96**

4.1 Order Statistics, 96
4.2 Multivariate Distributions, 98
4.3 Poisson Processes and Lifetimes, 100
4.4 Markov Processes, 104
4.5 Gaussian Processes, 105
4.6 Point Processes, 110
4.7 Metropolis' Method and Random Fields, 113
Exercises, 116

5 Variance Reduction **118**

5.1 Monte-Carlo Integration, 119
5.2 Importance Sampling, 122
5.3 Control and Antithetic Variates, 123
5.4 Conditioning, 134
5.5 Experimental Design, 137
Exercises, 139

6 Output Analysis **142**

6.1 The Initial Transient, 146
6.2 Batching, 150
6.3 Time-Series Methods, 155
6.4 Regenerative Simulation, 157
6.5 A Case Study, 161
Exercises, 169

7 Uses of Simulation **170**

7.1 Statistical Inference, 171
7.2 Stochastic Methods in Optimization, 178
7.3 Systems of Linear Equations, 186
7.4 Quasi-Monte-Carlo Integration, 189
7.5 Sharpening Buffon's Needle, 193
Exercises, 198

References **200**

Appendix A. Computer Systems **215**

Appendix B. Computer Programs **217**

B.1 Form $a \times b \bmod c$, 217
B.2 Check Primitive Roots, 219
B.3 Lattice Constants for Congruential Generators, 220
B.4 Test GFSR Generators, 227
B.5 Normal Variates, 228
B.6 Exponential Variates, 230
B.7 Gamma Variates, 230
B.8 Discrete Distributions, 231

Index **235**