

CONTENTS

PREFACE	xiii
CHAPTER 1 BACKGROUND MATERIAL	1
1.0 Introduction, <i>1</i>	
1.1 Some Subtleties in the Definition of Linearity, <i>2</i>	
1.2 Unilateral Laplace Transforms and a Generalized Initial-Value Theorem, <i>8</i>	
*1.3 Impulsive Functions, Signal Representations, and Input-Output Relations, <i>14</i>	
1.4 Some Remarks on the Use of Matrices, <i>27</i>	
CHAPTER 2 STATE-SPACE DESCRIPTIONS—SOME BASIC CONCEPTS	31
2.0 Introduction and Outline, <i>31</i>	
2.1 Some Canonical Realizations, <i>35</i>	
2.1.1 <i>Some remarks on analog computers, 35</i>	
2.1.2 <i>Four canonical realizations, 37</i>	
2.1.3 <i>Parallel and cascade realizations, 45</i>	

*Sections so marked throughout Contents may be skipped without loss of context.

- 2.2 State Equations in the Time and Frequency Domains, 49
 - 2.2.1 *Matrix notation and state-space equations, 50*
 - 2.2.2 *Obtaining state equations directly—some examples; linearization, 55*
 - 2.2.3 *A definition of state, 62*
 - 2.2.4 *More names and definitions, 66*
- 2.3 Initial Conditions for Analog-Computer Simulation; Observability and Controllability for Continuous- and Discrete-Time Realizations, 79
 - 2.3.1 *Determining the initial conditions; state observability, 80*
 - 2.3.2 *Setting up initial conditions; state controllability, 84*
 - 2.3.3 *Discrete-time systems; reachability and constructibility, 90*
 - *2.3.4 *Some worked examples, 103*
- 2.4 Further Aspects of Controllability and Observability, 120
 - 2.4.1 *Joint observability and controllability; the uses of diagonal forms, 120*
 - 2.4.2 *Standard forms for noncontrollable and/or nonobservable systems, 128*
 - 2.4.3 *The Popov-Belevitch-Hautus tests for controllability and observability, 135*
 - *2.4.4 *Some tests for relatively prime polynomials, 140*
 - *2.4.5 *Some worked examples, 145*
- *2.5 Solutions of State Equations and Modal Decompositions, 160
 - 2.5.1 *Time-invariant equations and matrix exponentials, 161*
 - 2.5.2 *Modes of oscillation and modal decompositions, 168*
- 2.6 A Glimpse of Stability Theory, 175
 - 2.6.1 *External and internal stability, 175*
 - 2.6.2 *The Lyapunov criterion, 177*
 - 2.6.3 *A stability result for linearized systems, 180*

CHAPTER 3 LINEAR STATE-VARIABLE FEEDBACK

187

- 3.0 Introduction, 187
- 3.1 Analysis of Stabilization by Output Feedback, 188
- 3.2 State-Variable Feedback and Modal Controllability, 197
 - 3.2.1 *Some formulas for the feedback gain, 198*
 - 3.2.2 *A transfer function approach, 202*
 - 3.2.3 *Some aspects of state-variable feedback, 204*

- *3.3 Some Worked Examples, 209
- 3.4 Quadratic Regulator Theory for Continuous-Time Systems, 218
 - 3.4.1 *Optimum steady-state solutions*, 219
 - *3.4.2 *Plausibility of the selection rule for the optimal poles*, 226
 - *3.4.3 *The algebraic Riccati equation*, 230
- 3.5 Discrete-Time Systems, 237
 - 3.5.1 *Modal controllability*, 238
 - 3.5.2 *Controllability to the origin, state-variable feedback, and the principle of optimality*, 239
 - *3.5.3 *The discrete-time quadratic regulator problem*, 243
 - *3.5.4 *Square-root and related algorithms*, 245

CHAPTER 4 ASYMPTOTIC OBSERVERS AND COMPENSATOR DESIGN 259

- 4.0 Introduction, 259
- 4.1 Asymptotic Observers for State Measurement, 260
- 4.2 Combined Observer-Controller Compensators, 268
- *4.3 Reduced-Order Observers, 281
- 4.4 An Optimality Criterion for Choosing Observer Poles, 293
- 4.5 Direct Transfer Function Design Procedures, 297
 - 4.5.1 *A transfer function reformulation of the observer-controller design*, 298
 - 4.5.2 *Some variants of the observer-controller design*, 304
 - 4.5.3 *Design via polynomial equations*, 306

CHAPTER 5 SOME ALGEBRAIC COMPLEMENTS 314

- 5.0 Introduction, 314
- 5.1 Abstract Approach to State-Space Realization Methods; Nerode Equivalence, 315
 - 5.1.1 *Realization from scalar transfer functions*, 315
 - 5.1.2 *Realization from the Markov parameters*, 322
 - *5.1.3 *Some algebraic language*, 326
- 5.2 Geometric Interpretation of Similarity Transformations; Linear Vector Spaces, 329
 - 5.2.1 *Vectors in n -space: linear independence*, 330
 - 5.2.2 *Matrices and transformations*, 333
 - 5.2.3 *Vector subspaces*, 338
 - 5.2.4 *Abstract linear vector spaces*, 341

**CHAPTER 6 STATE-SPACE AND MATRIX-FRACTION
DESCRIPTIONS OF MULTIVARIABLE
SYSTEMS****345**

- 6.0 Introduction, 345
- 6.1 Some Direct Realizations of Multivariable Transfer Functions, 346
- 6.2 State Observability and Controllability; Matrix-Fraction Descriptions, 352
 - 6.2.1 *The observability and controllability matrices, 353*
 - 6.2.2 *Standard forms for noncontrollable/nonobservable realizations; minimal realizations, 360*
 - 6.2.3 *Matrix-fraction descriptions, 367*
- 6.3 Some Properties of Polynomial Matrices, 372
 - 6.3.1 *Unimodular matrices; the Hermite form and coprime polynomial matrices, 373*
 - 6.3.2 *Column- and row-reduced matrices and some applications, 382*
 - 6.3.3 *The Smith form and related results, 390*
 - 6.3.4 *Linearizations, matrix pencils, and Kronecker forms, 393*
- 6.4 Some Basic State-Space Realizations, 403
 - 6.4.1 *Controller-form realizations from right MFDs, 403*
 - 6.4.2 *Some properties of the controller-form realization, 408*
 - 6.4.3 *Observer-form realizations from left MFDs, 413*
 - 6.4.4 *Controllability- and observability-form realizations, 417*
 - 6.4.5 *Canonical state-space realizations and canonical MFDs, 422*
 - 6.4.6 *Transformations of state-space realizations, 424*
- 6.5 Some Properties of Rational Matrices, 439
 - 6.5.1 *Irreducible MFDs and minimal realizations, 439*
 - 6.5.2 *The Smith-McMillan form of $H(s)$, 443*
 - 6.5.3 *Poles and zeros of multivariable transfer functions, 446*
 - 6.5.4 *Nullspace structure; minimal polynomial bases and Kronecker indices, 455*
- *6.6 Nerode Equivalence for Multivariable Systems, 470
- 6.7 Canonical Matrix-Fraction and State-Space Descriptions, 475
 - 6.7.1 *Hermite-form MFDs and Scheme I state-space realizations, 476*
 - 6.7.2 *Popov or polynomial-echelon MFDs and Scheme II realizations, 481*
 - *6.7.3 *The formal definition of canonical form, 492*

CHAPTER 7 STATE FEEDBACK AND COMPENSATOR DESIGN 499

- 7.0 Introduction, 499
- 7.1 State-Space Analysis of Linear State-Feedback, 499
 - 7.1.1 *Controller-form method*, 500
 - 7.1.2 *A direct method*, 503
 - *7.1.3 *The Brunovsky canonical form, Kronecker and factorization indices*, 505
- 7.2 Transfer Function Analysis of Linear State-Feedback, 506
 - 7.2.1 *Alternative formulas for the feedback gain matrix*, 507
 - *7.2.2 *Rosenbrock's control structure theorem*, 513
 - *7.2.3 *Two useful theorems on state and output feedback*, 518
- 7.3 Design of State Observers, 522
- 7.4 A Brief Look at the Multivariable Quadratic Regulator, 525
- 7.5 Transfer Function Design of Compensators, 532
- 7.6 Observability Under Feedback, and Invariant Zeros; $\{A,B\}$ -Invariant and Maximally Unobservable Subspaces, 540

CHAPTER 8 GENERAL DIFFERENTIAL SYSTEMS AND POLYNOMIAL MATRIX DESCRIPTIONS 549

- 8.0 Introduction, 549
- 8.1 Polynomial Matrix Descriptions and System Matrices, 551
- 8.2 State-Space Realizations of PMDs and Some Concepts of System Equivalence, 557
- 8.3 Some Properties and Applications of System Equivalence, 574
 - 8.3.1 *Some properties of irreducible PMDs*, 574
 - 8.3.2 *Poles and zeros of PMDs: transmission and decoupling zeros*, 577
 - 8.3.3 *Controllability and observability of interconnected systems*, 585

CHAPTER 9 SOME RESULTS FOR TIME-VARIANT SYSTEMS 594

- 9.0 Introduction, 594
- 9.1 Time-Variant State Equations; State-Transition Matrices, 595
- 9.2 Controllability and Observability Properties, 609
 - 9.2.1 *The controllability gramian*, 610
 - 9.2.2 *The observability gramian and a duality*, 615
- 9.3 Adjoint Systems and Some Applications, 622

CHAPTER 10 SOME FURTHER READING 632

- 10.0 Introduction, 632
- 10.1 Distributed Parameter Systems, 633
- 10.2 2-D Systems, 636
- 10.3 Some Other Applications of Algebraic Geometry;
Nonlinear Systems, 639
- 10.4 Approximation and Model Reduction, 639

APPENDIX: SOME FACTS FROM MATRIX THEORY 645

- 1 Basic Operations, 645
- 2 Some Determinant Formulas, 648
- 3 Block Matrices and Their Determinants, 650
- 4 Some Remarks on Linear Equations, 651
- 5 Some Results on Rank, 654
- 6 Some Formulas on Inverses, 655
- 7 Characteristic Polynomials and Resolvents, 656
- 8 The Cayley-Hamilton Theorem, 658
- 9 Companion Matrices, 659
- 10 Eigenvectors and Eigenvalues, 661
- 11 Spectral Decompositions and Matrix Diagonalization, 663
- 12 Similarity Transformations and Triangular Forms, 664
- 13 Defective Matrices and Jordan Forms, 665
- 14 Positive-Definite Matrices, 667
- 15 Singular Values of a Matrix, 667

INDEX 671