CONTENTS

VOLUME ONE

1	**Vectors in Classical Physics**	
	Introduction	1
1.1	Geometric and Algebraic Definitions of a Vector	1
1.2	The Resolution of a Vector into Components	3
1.3	The Scalar Product	4
1.4	Rotation of the Coordinate System: Orthogonal Transformations	5
1.5	The Vector Product	14
1.6	A Vector Treatment of Classical Orbit Theory	17
1.7	Differential Operations on Scalar and Vector Fields	19
*1.8	Cartesian-Tensors	33
2	**Calculus of Variations**	
	Introduction	43
2.1	Some Famous Problems	43
2.2	The Euler-Lagrange Equation	45
2.3	Some Famous Solutions	49
2.4	Isoperimetric Problems — Constraints	53
2.5	Application to Classical Mechanics	61
2.6	Extremization of Multiple Integrals	65
*2.7	Invariance Principles and Noether's Theorem	72
3	**Vectors and Matrices**	
	Introduction	85
3.1	Groups, Fields, and Vector Spaces	85
3.2	Linear Independence	89
3.3	Bases and Dimensionality	92
3.4	Isomorphisms	95
3.5	Linear Transformations	98
3.6	The Inverse of a Linear Transformation	100
3.7	Matrices	102
3.8	Determinants	109
3.9	Similarity Transformations	117

3.10	Eigenvalues and Eigenvectors	120
*3.11	The Kronecker Product	130
4	**Vector Spaces in Physics**	
	Introduction	142
4.1	The Inner Product	142
4.2	Orthogonality and Completeness	145
4.3	Complete Orthonormal Sets	148
4.4	Self-Adjoint (Hermitian and Symmetric) Transformations	151
4.5	Isometries—Unitary and Orthogonal Transformations	156
4.6	The Eigenvalues and Eigenvectors of Self-Adjoint and Isometric Transformations	158
4.7	Diagonalization	164
4.8	On the Solvability of Linear Equations	171
4.9	Minimum Principles	175
4.10	Normal Modes	184
4.11	Perturbation Theory—Nondegenerate Case	192
*4.12	Perturbation Theory—Degenerate Case	198
5	**Hilbert Space—Complete Orthonormal Sets of Functions**	
	Introduction	212
5.1	Function Space and Hilbert Space	213
5.2	Complete Orthonormal Sets of Functions	217
5.3	The Dirac δ-Function	224
5.4	Weierstrass's Theorem: Approximation by Polynomials	228
5.5	Legendre Polynomials	233
5.6	Fourier Series	239
5.7	Fourier Integrals	246
5.8	Spherical Harmonics and Associated Legendre Functions	253
5.9	Hermite Polynomials	261
5.10	Sturm-Liouville Systems—Orthogonal Polynomials	263
5.11	A Mathematical Formulation of Quantum Mechanics	277
	VOLUME TWO	
6	**Elements and Applications of the Theory of Analytic Functions**	
	Introduction	305
6.1	Analytic Functions—The Cauchy-Riemann Conditions	306
6.2	Some Basic Analytic Functions	312
6.3	Complex Integration—The Cauchy-Goursat Theorem	322
6.4	Consequences of Cauchy's Theorem	330
6.5	Hilbert Transforms and the Cauchy Principal Value	335
6.6	An Introduction to Dispersion Relations	340
6.7	The Expansion of an Analytic Function in a Power Series	349
6.8	Residue Theory—Evaluation of Real Definite Integrals and Summation of Series	358
6.9	Applications to Special Functions and Integral Representations	371

7 Green's Functions

Introduction 388
7.1 A New Way to Solve Differential Equations 388
7.2 Green's Functions and Delta Functions 395
7.3 Green's Functions in One Dimension 401
7.4 Green's Functions in Three Dimensions 411
7.5 Radial Green's Functions 420
7.6 An Application to the Theory of Diffraction 433
7.7 Time-dependent Green's Functions: First Order 442
7.8 The Wave Equation 453

8 Introduction to Integral Equations

Introduction 469
8.1 Iterative Techniques—Linear Integral Operators 469
8.2 Norms of Operators 474
8.3 Iterative Techniques in a Banach Space 479
8.4 Iterative Techniques for Nonlinear Equations 484
8.5 Separable Kernels 489
8.6 General Kernels of Finite Rank 496
8.7 Completely Continuous Operators 503

9 Integral Equations in Hilbert Space

Introduction 518
9.1 Completely Continuous Hermitian Operators 518
9.2 Linear Equations and Perturbation Theory 531
9.3 Finite-Rank Techniques for Eigenvalue Problems 541
9.4 The Fredholm Alternative For Completely Continuous Operators . . . 549
9.5 The Numerical Solution of Linear Equations 555
9.6 Unitary Transformations 563

10 Introduction to Group Theory

Introduction 580
10.1 An Inductive Approach 580
10.2 The Symmetric Groups 586
10.3 Cosets, Classes, and Invariant Subgroups 592
10.4 Symmetry and Group Representations 599
10.5 Irreducible Representations 604
10.6 Unitary Representations, Schur's Lemmas, and Orthogonality Relations 610
10.7 The Determination of Group Representations 622
10.8 Group Theory in Physical Problems 633

General Bibliography 649

Index to Volume One 651

Index to Volume Two 657