

Contents

Chapter 1. Introduction	1
1.1. Space barriers and divisibility barriers	1
1.2. Tetrahedron packings	3
1.3. A multipartite Hajnal-Szemerédi theorem	4
1.4. Algorithmic aspects of hypergraph matchings	5
1.5. Notation	6
Chapter 2. Results and examples	7
2.1. Almost perfect matchings	7
2.2. Partite systems	9
2.3. Lattice-based constructions	10
2.4. Perfect matchings	12
2.5. Further results	15
2.6. Outline of the proofs	16
Chapter 3. Geometric Motifs	19
Chapter 4. Transferrals	23
4.1. Irreducibility	23
4.2. Transferral digraphs	24
4.3. Completion of the transferral digraph	28
Chapter 5. Transferrals via the minimum degree sequence	31
Chapter 6. Hypergraph Regularity Theory	39
6.1. Hypergraph regularity	39
6.2. The Regular Approximation Lemma	40
6.3. The hypergraph blowup lemma	41
6.4. Reduced k -systems	42
6.5. Proof of Lemma 5.5	45
Chapter 7. Matchings in k -systems	51
7.1. Fractional perfect matchings	51
7.2. Almost perfect matchings	57
7.3. Perfect matchings	61
Chapter 8. Packing Tetrahedra	69
8.1. Packing to within a constant	71
8.2. Properties of index vectors	72
8.3. Divisibility barriers with two parts	73
8.4. Divisibility barriers with more parts	78

8.5. The main case of Theorem 1.1	81
8.6. The case when 8 divides n	84
8.7. Strong stability for perfect matchings	85
Chapter 9. The general theory	89
Acknowledgements	91
Bibliography	93