

Contents

<i>Foreword</i>	vii
<i>Preface</i>	ix
<i>Acknowledgements</i>	xiii
<i>About the Authors</i>	xv
<i>List of Figures</i>	xxv
<i>List of Tables</i>	xxvii
1. Introduction	1
1.1 Questions about Model Uncertainty	1
1.2 Ten Papers about Model Uncertainty	8
2. Discounted Linear Exponential Quadratic Gaussian Control	23
2.1 Cost Formulation	23
2.2 Cost Recursions and Aggregator Functions	24
2.3 Infinite Horizon Costs	26
2.4 Arbitrary Time-invariant Linear Control Laws	27
2.5 Solution to the Infinite Horizon Discounted Problem	29
2.6 Summary	32
3. Robust Permanent Income and Pricing	33
<i>with Thomas D. Tallarini</i>	
3.1 Introduction	33
3.2 Recursive Risk Sensitive Control	36

3.3	Robust Permanent Income Theory	42
3.4	Estimation	49
3.5	Asset Pricing	57
3.6	Quantifying Robustness from the Market Price of Risk	62
3.7	Intertemporal Mean-risk Trade-offs	69
3.8	Conclusions	75
Appendix 3.A	Subgradient Inequality	77
Appendix 3.B	Computing Prices for State-contingent Utility	78
Appendix 3.C	Computing the Conditional Variance of the Stochastic Discount Factor	79
4.	A Quartet of Semigroups for Model Specification, Robustness, Prices of Risk, and Model Detection	83
	<i>with Evan W. Anderson</i>	
4.1	Introduction	83
4.2	Overview	88
4.3	Mathematical Preliminaries	91
4.4	A Tour of Four Semigroups	97
4.5	Model Misspecification and Robust Control	100
4.6	Portfolio Allocation	107
4.7	Pricing Risky Claims	112
4.8	Statistical Discrimination	118
4.9	Entropy and the Market Price of Uncertainty	129
4.10	Concluding Remarks	140
Appendix 4.A	Proof of Theorem 4.5.1	142
5.	Robust Control and Model Uncertainty	145
5.1	Introduction	145
5.2	A Benchmark Resource Allocation Problem	146
5.3	Model Misspecification	146
5.4	Two Robust Control Problems	148
5.5	Recursivity of the Multiplier Formulation	149
5.6	Two Preference Orderings	150
5.7	Recursivity of the Preference Orderings	152
5.8	Concluding Remarks	153

6. Robust Control and Model Misspecification	155
<i>with Gauhar A. Turmuhambetova and Noah Williams</i>	
6.1 Introduction	155
6.2 Overview	159
6.3 Three Ordinary Control Problems	165
6.4 Fear of Model Misspecification	170
6.5 Two Robust Control Problems Defined on Sets of Probability Measures	171
6.6 Games on Fixed Probability Spaces	180
6.7 Sequential Timing Protocol for a Penalty Formulation	186
6.8 Sequential Timing Protocol for a Constraint Formulation	190
6.9 A Recursive Multiple Priors Formulation	197
6.10 Concluding Remarks	201
Appendix 6.A Cast of Characters	203
Appendix 6.B Discounted Entropy	204
Appendix 6.C Absolute Continuity of Solutions	208
Appendix 6.D Three Ways to Verify Bellman–Isaacs Condition	210
Appendix 6.E Recursive Stackelberg Game and Bayesian Problem	213
7. Doubts or Variability?	217
<i>with Francisco Barillas</i>	
7.1 Introduction	217
7.2 The Equity Premium and Risk-free Rate Puzzles	219
7.3 The Choice Setting	222
7.4 A Type I Agent: Kreps–Porteus–Epstein– Zin–Tallarini	225
7.5 A Type I Agent Economy with High Risk Aversion Attains HJ Bound	227
7.6 Reinterpretations	228
7.7 Reinterpreting Tallarini	238
7.8 Welfare Gains from Eliminating Model Uncertainty	243

7.9	Dogmatic Bayesians and Learning	251
7.10	Concluding Remarks	253
	Appendix 7.A Formulas for Trend Stationary Model	254
8.	Robust Estimation and Control	
	without Commitment	257
8.1	Introduction	257
8.2	A Control Problem without Model Uncertainty	261
8.3	Using Martingales to Represent Model Misspecifications	265
8.4	Two Pairs of Operators	267
8.5	Control Problems with Model Uncertainty	270
8.6	The $\theta_1 = \theta_2$ Case	276
8.7	Implied Worst Case Model of Signal Distortion	284
8.8	A Recursive Multiple Priors Model	286
8.9	Risk Sensitivity and Compound Lotteries	287
8.10	Another Example	288
8.11	Concluding Remarks	290
9.	Fragile Beliefs and the Price of Uncertainty	293
9.1	Introduction	293
9.2	Stochastic Discounting and Risks	296
9.3	Three Information Structures	301
9.4	Risk Prices	303
9.5	A Full-Information Perspective on Agents' Learning	304
9.6	Price Effects of Concerns About Robustness	307
9.7	Illustrating the Mechanism	315
9.8	Concluding Remarks	327
	Appendix 9.A Detection Error Probabilities	329
10.	Beliefs, Doubts and Learning: Valuing Macroeconomic Risk	331
	<i>by Lars Peter Hansen</i>	
10.1	Introduction	331
10.2	Rational Expectations and Econometrics	333
10.3	Statistical Precision	338
10.4	Risk Prices and Statistical Ambiguity	343
10.5	Statistical Challenges	347
10.6	Learning	352

10.7	Beliefs and Preferences	360
10.8	Learning and Uncertainty Premia	365
10.9	Extensions	375
10.10	Conclusion	376
11.	Three Types of Ambiguity	379
11.1	Illustrative Model	385
11.2	No Concern about Robustness	386
11.3	Representing Probability Distortions	392
11.4	The First Type of Ambiguity	394
11.5	Heterogeneous Beliefs without Robustness	401
11.6	The Second Type of Ambiguity	408
11.7	The Third Type of Ambiguity	409
11.8	Comparisons	412
11.9	Numerical Example	418
11.10	Concluding Remarks	424
Appendix 11.A	Some Basic Proofs	425
Appendix 11.B	Example without Robustness	426
Appendix 11.C	Example with First Type of Ambiguity	428
Appendix 11.D	Sensitivity to Robustness	430
	<i>Bibliography</i>	431
	<i>Author Index</i>	447
	<i>Subject Index</i>	453