

Contents

	<i>Preface</i>	<i>page xi</i>
	<i>List of symbols</i>	<i>xiii</i>
1	Introduction and message of the book	1
	1.1 Why polynomial optimization?	1
	1.2 Message of the book	3
	1.3 Plan of the book	8
	PART I POSITIVE POLYNOMIALS AND MOMENT PROBLEMS	13
2	Positive polynomials and moment problems	15
	2.1 Sum of squares representations and semidefinite optimization	16
	2.2 Representation theorems: univariate case	19
	2.3 Representation theorems: multivariate case	22
	2.4 Polynomials positive on a compact basic semi-algebraic set	25
	2.5 Polynomials nonnegative on real varieties	32
	2.6 Representations with sparsity properties	34
	2.7 Moment problems	36
	2.8 Exercises	49
	2.9 Notes and sources	50
3	Another look at nonnegativity	54
	3.1 Nonnegativity on closed sets	54
	3.2 The compact case	55
	3.3 The noncompact case	58
	3.4 A symmetric duality principle	60

3.5	Exercises	61
3.6	Notes and sources	61
4	The cone of polynomials nonnegative on \mathbf{K}	62
4.1	Introduction	62
4.2	Inner semidefinite approximations when \mathbf{K} is compact	62
4.3	Outer semidefinite approximations	64
4.4	Approximations of the dual cone	69
4.5	The cone of copositive matrices and its dual	71
4.6	Exercises	76
4.7	Notes and sources	77
	PART II POLYNOMIAL AND SEMI-ALGEBRAIC OPTIMIZATION	79
5	The primal and dual points of view	81
5.1	Polynomial optimization as an infinite-dimensional LP	82
5.2	Polynomial optimization as a finite-dimensional convex optimization problem	84
5.3	Exercises	86
6	Semidefinite relaxations for polynomial optimization	87
6.1	Constrained polynomial optimization	87
6.2	Discrete optimization	103
6.3	Unconstrained polynomial optimization	106
6.4	Exercises	113
6.5	Notes and sources	114
7	Global optimality certificates	116
7.1	Putinar versus Karush–Kuhn–Tucker	116
7.2	Krivine–Stengle versus Fritz John	124
7.3	Exercises	128
7.4	Notes and sources	129
8	Exploiting sparsity or symmetry	130
8.1	Exploiting sparsity	130
8.2	Exploiting symmetry	133
8.3	Exercises	135
8.4	Notes and sources	136
9	LP-relaxations for polynomial optimization	137
9.1	LP-relaxations	138
9.2	Interpretation and the dual method in NLP	141

9.3	Contrasting semidefinite and LP-relaxations	143
9.4	An intermediate hierarchy of convex relaxations	145
9.5	Exercises	148
9.6	Notes and sources	148
10	Minimization of rational functions	150
10.1	Minimizing a rational function	150
10.2	Minimizing a sum of many rational functions	153
10.3	Exercises	162
10.4	Notes and sources	163
11	Semidefinite relaxations for semi-algebraic optimization	164
11.1	Introduction	164
11.2	Semi-algebraic functions	165
11.3	A Positivstellensatz for semi-algebraic functions	168
11.4	Optimization of semi-algebraic functions	170
11.5	Exercises	172
11.6	Notes and sources	173
12	Polynomial optimization as an eigenvalue problem	174
12.1	A converging sequence of upper bounds	175
12.2	The associated eigenvalue problem	186
12.3	On copositive programs	188
12.4	Exercises	192
12.5	Notes and sources	193
	PART III SPECIALIZATIONS AND EXTENSIONS	195
13	Convexity in polynomial optimization	197
13.1	Convexity and polynomials	197
13.2	Semidefinite representation of convex sets	210
13.3	Convex polynomial programs	214
13.4	Exercises	218
13.5	Notes and sources	219
14	Parametric optimization	221
14.1	Introduction	221
14.2	Parametric optimization	222
14.3	On robust polynomial optimization	233
14.4	A “joint+marginal” algorithm in optimization	236
14.5	Exercises	240
14.6	Notes and sources	241

15	Convex underestimators of polynomials	243
	15.1 Introduction	243
	15.2 Convex polynomial underestimators	244
	15.3 Comparison with the $\alpha\mathbf{BB}$ convex underestimator	249
	15.4 Exercises	254
	15.5 Notes and sources	255
16	Inverse polynomial optimization	257
	16.1 Introduction	257
	16.2 Computing an inverse optimal solution	258
	16.3 A canonical form for the ℓ_1 -norm	267
	16.4 Exercises	270
	16.5 Notes and sources	271
17	Approximation of sets defined with quantifiers	272
	17.1 Introduction	272
	17.2 Inner and outer approximations	274
	17.3 Extensions	279
	17.4 Exercises	284
	17.5 Notes and sources	285
18	Level sets and a generalization of the Löwner–John problem	286
	18.1 Introduction	286
	18.2 Quasi-homogeneous polynomials and their level sets	286
	18.3 A generalization of the Löwner–John problem	292
	18.4 A numerical scheme	299
	18.5 Exercises	303
	18.6 Notes and sources	304
<i>Appendix A</i>	Semidefinite programming	306
<i>Appendix B</i>	The GloptiPoly software	309
	<i>References</i>	324
	<i>Index</i>	337