

Contents

<i>Foreword by Bjarne Toft</i>	<i>page</i> xiii
<i>Preface</i>	xv
Preliminaries	1
LOWELL W. BEINEKE and ROBIN J. WILSON	
1. Graph theory	1
2. Graph colourings	9
1 Colouring graphs on surfaces	13
BOJAN MOHAR	
1. Introduction	13
2. Planar graphs are 4-colourable and 5-choosable	14
3. Heawood's formula	18
4. Colouring with few colours	20
5. Grötzsch's theorem and its generalizations	23
6. Colouring-flow duality	25
7. The acyclic chromatic number	29
8. Degenerate colourings	30
9. The star chromatic number	31
10. Summary	32
2 Brooks's theorem	36
MICHAEL STIEBITZ and BJARNE TOFT	
1. Introduction	36
2. Proofs of Brooks's theorem	37
3. Critical graphs with few edges	41
4. Bounding χ by Δ and ω	45
5. Graphs with χ close to Δ	48
6. Notes	50

3	Chromatic polynomials	56
	BILL JACKSON	
	1. Introduction	56
	2. Definitions and elementary properties	57
	3. Log concavity and other inequalities	59
	4. Chromatic roots	60
	5. Related polynomials	64
4	Hadwiger's conjecture	73
	KEN-ICHI KAWARABAYASHI	
	1. Introduction	73
	2. Complete graph minors: early results	74
	3. Contraction-critical graphs	75
	4. Algorithmic aspects of the weak conjecture	79
	5. Algorithmic aspects of the strong conjecture	81
	6. The odd conjecture	82
	7. Independent sets and Hadwiger's conjecture	85
	8. Other variants of the conjecture	86
	9. Open problems	89
5	Edge-colourings	94
	JESSICA MCDONALD	
	1. Introduction	94
	2. Elementary sets and Kempe changes	96
	3. Tashkinov trees and upper bounds	97
	4. Towards the Goldberg–Seymour conjecture	101
	5. Extreme graphs	103
	6. The classification problem and critical graphs	105
	7. The dichotomy of edge-colouring	108
	8. Final thoughts	109
6	List-colourings	114
	MICHAEL STIEBITZ and MARGIT VOIGT	
	1. Introduction	114
	2. Orientations and list-colourings	118
	3. Planar graphs	121
	4. Precolouring extensions	128
	5. Notes	129
7	Perfect graphs	137
	NICOLAS TROTIGNON	
	1. Introduction	137

2.	Lovász's perfect graph theorem	139
3.	Basic graphs	141
4.	Decompositions	142
5.	The strategy of the proof	146
6.	Book from the Proof	148
7.	Recognizing perfect graphs	151
8.	Berge trigraphs	152
9.	Even pairs: a shorter proof of the SPGT	154
10.	Colouring perfect graphs	155
8	Geometric graphs	161
	ALEXANDER SOIFER	
1.	The chromatic number of the plane	161
2.	The polychromatic number: lower bounds	162
3.	The de Bruijn–Erdős reduction to finite sets	165
4.	The polychromatic number: upper bounds	167
5.	The continuum of 6-colourings	169
6.	Special circumstances	171
7.	Space explorations	172
8.	Rational spaces	173
9.	One odd graph	175
10.	Influence of set theory axioms	175
11.	Predicting the future	177
9	Integer flows and orientations	181
	HONGJIAN LAI, RONG LUO and CUN-QUAN ZHANG	
1.	Introduction	181
2.	Basic properties	183
3.	4-flows	185
4.	3-flows	185
5.	5-flows	187
6.	Bounded orientations and circular flows	188
7.	Modulo orientations and $(2 + 1/t)$ -flows	190
8.	Contractible configurations	191
9.	Related problems	194
10	Colouring random graphs	199
	ROSS J. KANG and COLIN MCDIARMID	
1.	Introduction	199
2.	Dense random graphs	202
3.	Sparse random graphs	208
4.	Random regular graphs	214

	5. Random geometric graphs	217
	6. Random planar graphs and related classes	219
	7. Other colourings	222
11	Hypergraph colouring	230
	CSILLA BUJTÁS, ZSOLT TUZA and VITALY VOLOSHIN	
	1. Introduction	230
	2. Proper vertex- and edge-colourings	234
	3. \mathcal{C} -colourings	238
	4. Colourings of mixed hypergraphs	243
	5. Colour-bounded and stably bounded hypergraphs	247
	6. Conclusion	251
12	Chromatic scheduling	255
	DOMINIQUE DE WERRA and ALAIN HERTZ	
	1. Introduction	255
	2. Colouring with weights on the vertices	256
	3. List-colouring	258
	4. Mixed graph colouring	260
	5. Co-colouring	261
	6. Colouring with preferences	262
	7. Bandwidth colouring	264
	8. Edge-colouring	266
	9. Sports scheduling	268
	10. Balancing requirements	269
	11. Compactness	273
	12. Conclusion	274
13	Graph colouring algorithms	277
	THORE HUSFELDT	
	1. Introduction	277
	2. Greedy colouring	279
	3. Recursion	284
	4. Subgraph expansion	287
	5. Local augmentation	289
	6. Vector colouring	292
	7. Reductions	296
	8. Conclusion	301
14	Colouring games	304
	ZSOLT TUZA and XUDING ZHU	
	1. Introduction	304

2. Marking games	307
3. Greedy colouring games	312
4. Playing on the edge-set	312
5. Oriented and directed graphs	313
6. Asymmetric games	315
7. Relaxed games	316
8. Paintability	316
9. Achievement and avoidance games	321
10. The acyclic orientation game	322
15 Unsolved graph colouring problems	327
TOMMY JENSEN and BJARNE TOFT	
1. Introduction	327
2. Complete graphs and chromatic numbers	328
3. Graphs on surfaces	333
4. Degrees and colourings	339
5. Edge-colourings	344
6. Flow problems	348
7. Concluding remarks	350
<i>Notes on contributors</i>	358
<i>Index</i>	363